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AN EXTENSION OF MICHAEL'S SELECTION THEOREM

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Abstract. In this paper, we extend earlier Michael's selection theorem in the general nonconvex setting. Indeed, we deal with the case of almost lower semicontinuous multifunctions T on a zero-dimensional paracompact topological space X. Based on a kind of equicontinuous property, we prove that any *a.l.s.c.* *ECP* multifunction from X into a complete metric space Y still has a continuous selection.

1. Introduction and preliminaries

A *paracompact space* is a topological space in which every open covering admits an open locally finite refinement. A topological space X is said to be *zero-dimensional*, denoted by $\dim X = 0$, if every finite open covering of X has a disjoint finite open refinement. The following Michael's selection theorem [10, Theorem 2] might be considered as a starting point of the present paper:

Theorem 1.1. *If X is paracompact and zero-dimensional, Y a complete metric space, and $T : X \longrightarrow 2^Y$ is a l.s.c. multifunction such that $T(x)$ is closed for all $x \in X$, then T admits a continuous selection $f : X \longrightarrow Y$; that is, f is a continuous single-valued function such that $f(x) \in T(x)$ for all $x \in X$.*

Continuous selection plays an important role in optimization theory, especially in the proof of existence of fixed points for a multifunction, see for example [1, 5, 6, 9, 12, 13]. In this paper, we shall extend the above Michael's selection theorem to almost lower semicontinuous multifunctions, which are weaker than usual lower semicontinuity. Beyond the convexity and compactness, the results derived here generalize various earlier ones from classic continuous selection theory, as will be indicated below.

For any set Y, let 2^Y denote the collection of all nonempty subsets of Y. When Y is a metric space with a metric d, we may define the $\epsilon-$neighborhood of a subset A of Y by

$$B_\epsilon(A) := \{y \in Y \mid d(y, A) < \epsilon\},$$

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where $d(y, A) := \inf\{d(x, y) \mid x \in A\}$. The *closure* and *interior* of A shall be denoted by $\overline{A}$ and int$A$, respectively.

A *multifunction* T from X to Y is simply a function which assigns each point x of X to a (possibly empty) subset $T(x)$ of Y. We shall say that T is *lower semicontinuous* (*l.s.c.*) at x, provided for any open set G, with $G \cap T(x) \neq \emptyset$, there is a neighborhood V_x of x such that $G \cap T(z) \neq \emptyset$ for all $z \in V_x$. T is *lower semicontinuous*, if T is *l.s.c.* at each $x \in X$. Unfortunately, lower semicontinuity is not necessarily often absent in many optimization problems. This has forced many researchers to employ some weaker continuity, such as [2, 5, 7, 11, 12]. A multifunction $T : X \longrightarrow 2^Y$ is *almost lower semicontinuous* (*a.l.s.c.*) at x, if for any $\epsilon > 0$, there exists a neighborhood V_x of x such that

$$\bigcap_{z \in V_x} B_\epsilon(T(z)) \neq \emptyset.$$

T is *almost lower semicontinuous*, if it is *a.l.s.c.* at each x. It should be noted that an *a.l.s.c.* multifunction does not have a continuous selection in general; for example, let $T : R \longrightarrow 2^{R^2}$ be defined by

$$T(x) := \begin{cases} \{(t, xt) \mid 0 \leq t \leq 1\} & \text{, if } x \text{ is irrational;} \\ \{(t, 0) \mid 0 \leq t \leq 1\} & \text{, if } x \text{ is rational and } x \neq 0; \\ \{(1, 0)\} & \text{, if } x = 0 \text{ .} \end{cases}$$

Then such a multifunction T is *a.l.s.c.*, but T has no continuous selection (see [11]).

Given a multifunction $T : X \longrightarrow 2^Y$, we call a single-valued function $f : X \longrightarrow Y$ to be an $\epsilon-$*approximate selection* for T, provided that $f(x) \in B_\epsilon(T(x))$ for all $x \in X$. For each $\epsilon > 0$, we define

$$C_\epsilon(T) := \{f : X \longrightarrow Y \mid f \text{ is continuous, and } f(x) \in B_\epsilon(T(x)),\ \forall\, x \in X\}.$$

Under this notion, we shall say that $T : X \longrightarrow 2^Y$ is an *ECP multifunction*, if it has the equicontinuous property : for any $x \in X$ and $\epsilon > 0$, there exist $\sigma := \sigma(\epsilon) > 0$, and a neighborhood N_x of x, such that

(1) $d(f(z), f(x)) < \frac{\sigma}{2},\ \forall\, f \in C_{\frac{\sigma}{2}}(T),\ \forall\, z \in N_x$;
(2) $d(y_1, y_2) < \epsilon,\ \forall\, y_1, y_2 \in \bigcap_{z \in N_x} B_\sigma(T(z))$.

Recently, we proved the following selection theorem [4, Theorem 3.6] for an *a.l.s.c.* C-set-valued multifunction:

Theorem 1.2. *Let X be paracompact, Y a complete LC-metric space, Z a closed subset of X, with* $\dim_X Z \leq 0$, *and $T : X \longrightarrow 2^Y$ be a multifunction. If there is an a.l.s.c. ECP multifunction $S : X \longrightarrow 2^Y$ satisfying*

(1) *$S(x)$ is a C-set for all $x \in X \setminus Z$, closed for all $x \in X$, and $S(x) \subset T(x),\ \forall\, x \in X$,*
(2) *for each $x \in X$, $\overline{B}_\eta(T(x))$ is compact for some $\eta > 0$,*
(3) *S has the one-point extension property,*

then T admits a continuous selection.

As a special case where $Z = X$, we have the following result at once.

Corollary 1.3. *Let X be paracompact, with* $\dim X = 0$*, Y a complete LC-metric space, and $T : X \longrightarrow 2^Y$ be a multifunction. If there is an a.l.s.c. ECP multifunction $S : X \longrightarrow 2^Y$ satisfying*

(1) *each $S(x)$ is closed and $S(x) \subset T(x)$, $\forall\ x \in X$,*
(2) *for each $x \in X$, $\overline{B}_\eta(T(x))$ is compact for some $\eta > 0$,*
(3) *S has the one-point extension property,*

then T admits a continuous selection.

Here, we say that a multifunction $S : X \longrightarrow 2^Y$ has the *one-point extension property*, provided that for each *l.s.c.* multifunction $L : X \longrightarrow 2^Y$ with $L(z) \subset S(z)$, $\forall\ z \in X$, and for each $(x, a) \in G(S) \setminus G(L)$ (i.e., the point (x, a) lies on the graph of S, but not on the graph of L), there is a *l.s.c.* multifunction $L^* : X \longrightarrow 2^Y$ such that $(x, a) \in G(L^*)$ and

$$L(z) \subset L^*(z) \subset S(z),\ \forall\ z \in X.$$

In the sequel, we shall show that the so-called "one-point extension property" in Corollary 1.3 is redundant. In order to develop our main result, let us begin with listing some basic known lemmas, which will be used in the next section.

Lemma 1.4. *Let X be a topological space, and (Y, d) be a metric space. Then a multifunction $T : X \longrightarrow 2^Y$ is l.s.c. at $x \in X$ if, and only if, for each $\epsilon > 0$ and $y \in T(x)$, there exists a neighborhood V_x of x such that $y \in B_\epsilon(T(z))$ for all $z \in V_x$.*

Lemma 1.5. *Let (Y, d) be a metric space, $A \subset Y$, and $\alpha > 0$, $\beta > 0$. Then*

(1) $\overline{B}_\alpha(\overline{B}_\beta(A)) \subset \overline{B}_{\alpha+\beta}(A)$;
(2) $\overline{B}_\alpha(B_\beta(A)) \subset B_{\alpha+\beta}(A)$.

Lemma 1.6. *Let $S : X \longrightarrow 2^Y$ and $S_i : X \longrightarrow 2^Y$ be l.s.c. for each $i \in I$.*

(1) *$\overline{S}$ is l.s.c.. (Here, $\overline{S}(x) := \overline{S(x)}$, $\forall\ x \in X$.)*
(2) *The multifunction $T : X \longrightarrow 2^Y$, defined by $T(x) := \bigcup_{i \in I} S_i(x)$, $\forall\ x \in X$, is l.s.c..*
(3) *If $\alpha : A \longrightarrow Y$ is a continuous selection of S on a nonempty closed subset A of X, that is, $\alpha(x) \in S(x)$ for all $x \in A$, then the following multifunction $T : X \longrightarrow 2^Y$ is l.s.c., where*

$$T(x) := \begin{cases} S(x) & , \text{ if } x \in X \setminus A, \\ \{\alpha(x)\} & , \text{ if } x \in A. \end{cases}$$

2. Main Selection Theorem

In order to develop our main existence results, we assume throughout this section that X is paracompact, with $\dim X = 0$, and Y is a metric space. First, we establish a key existence of an $\epsilon-$approximate selection for *a.l.s.c.* multifunctions.

Theorem 2.1. *If $T : X \longrightarrow 2^Y$ is an a.l.s.c. multifunction, then for any $\epsilon > 0$, T has a continuous $\epsilon-$approximate selection.*

Proof. The proof goes along the line of [10, Theorem 2] with some changes pertaining to the present contex. For a fixed $\epsilon > 0$, we define $U_y := \{x \in X \mid y \in B_\epsilon(T(x))\}$ for each $y \in Y$. Since T is *a.l.s.c.* at each $x \in X$, there is a neighborhood W_x of x such that $\bigcap_{z\in W_x} B_\epsilon(T(z)) \neq \emptyset$, say $y_x \in B_\epsilon(T(z))$ for all $z \in W_x$. That is,

$$x \in W_x \subset \{z \in X \mid y_x \in B_\epsilon(T(z))\}.$$

Consequently, we have $x \in \text{int}U_y$ for some $y \in Y$, and hence $\{\text{int}U_y \mid y \in Y\}$ is an open cover of X. Since X is paracompact and zero-dimensional, there exists a locally finite disjoint open cover $\mathcal{V}$ of X, which is a refinement of $\{\text{int}U_y \mid y \in Y\}$. Thus, for each $V \in \mathcal{V}$, we can pick one $y^V \in Y$ such that $V \subset \text{int}U_{y^V}$, and define a single-valued function $f : X \longrightarrow Y$ by $f(x) = y^V$ for all $x \in V$. Then f is clearly continuous, as shown in [10].

Now, we verify that $f(x) \in B_\epsilon(T(x))$ for all $x \in X$. For each $x \in X$, there exists a unique $V \in \mathcal{V}$ such that $x \in V$. It follows that $x \in \text{int}U_{y^V} \subset U_{y^V}$, and hence,

$$f(x) = y^V \in B_\epsilon(T(x)).$$

This shows that f is a continuous $\epsilon-$approximate selection of T. □

According to Theorem 2.1, we remark that for each $\epsilon > 0$ and for any *a.l.s.c.* multifunction $T : X \longrightarrow 2^Y$, the set

$$C_\epsilon(T) := \{f : X \longrightarrow Y \mid f \text{ is continuous, and } f(x) \in B_\epsilon(T(x)),\ \forall\ x \in X\}$$

is not empty.

Proposition 2.2. *For each $\epsilon > 0$, the multifunction $T_\epsilon : X \longrightarrow 2^Y$, defined by*

$$T_\epsilon(x) := \{f(x) \mid f \in C_\epsilon(T)\},\ \forall\ x \in X,$$

is l.s.c.. In particular, $T_\epsilon(x) \neq \emptyset$ for all $x \in X$.

Proof. Since $C_\epsilon(T) \neq \emptyset$ for all $\epsilon > 0$, it follows that $T_\epsilon(x) \neq \emptyset$ for all $x \in X$. For any open set G, with $G \cap T_\epsilon(x) \neq \emptyset$, there exists $f \in C_\epsilon(T)$ such that $f(x) \in G$, which implies $x \in f^{-1}(G)$. Since f is continuous, $f^{-1}(G)$ is open, and hence, there is a neighborhood N_x of x such that $x \in N_x \subset f^{-1}(G)$. Thus, for any $z \in N_x$, we have $f(z) \in G$. Consequently,

$$T_\epsilon(z) \bigcap G \neq \emptyset,\ \forall\ z \in N_x.$$

This shows that T_ϵ is *l.s.c.* at x. Therefore, we complete the proof. □

We remark that both sets $C_\epsilon(T)$ and $T_\epsilon(x)$ are not empty, whenever T is an *a.l.s.c.* multifunction. Also, by the definitions of $C_\epsilon(T)$ and T_ϵ, we have

(1) if $\epsilon_1 \leq \epsilon_2$, then $C_{\epsilon_1}(T) \subset C_{\epsilon_2}(T)$, and $T_{\epsilon_1}(x) \subset T_{\epsilon_2}(x),\ \forall\ x \in X$;
(2) $T_\epsilon(x) \subset B_\epsilon(T(x)),\ \forall\ x \in X$.

Proposition 2.3. *Let Y be a complete metric space. If for each $x \in X$, there exists some $\eta := \eta(x) > 0$ such that $\overline{B}_\eta(T(x))$ is compact, then for any $x \in X$ and $\epsilon > 0$, there is $\delta := \delta(x, \epsilon) > 0$ such that $T_\delta(x) \subset B_{\frac{\epsilon}{2}}(T_0(x))$, where*

$$T_0(x) := \bigcap_{\epsilon>0} T_\epsilon(x),\ \forall\ x \in X.$$

Moreover, $T_0(x) \neq \emptyset,\ \forall\ x \in X$.

Proof. For contradiction, we assume that there exist some $x_0 \in X$ and $\epsilon_0 > 0$ such that for any $\delta > 0$, we always have

$$T_\delta(x_0) \not\subset B_{\frac{\epsilon_0}{2}}(T_0(x_0)).$$

For this x_0, there exists $\eta > 0$ such that $\overline{B}_\eta(T(x_0))$ is compact. Now, we take a sequence $\{\delta_n\}_{n=1}^\infty$ with $\delta_1 = \eta$ and $\delta_n \downarrow 0$ as $n \to \infty$ such that

$$T_{\delta_n}(x_0) \not\subset B_{\frac{\epsilon_0}{2}}(T_0(x_0)),$$

i.e., for each n there exists y_n such that

$$y_n \in T_{\delta_n}(x_0),\ \text{but}\ \ y_n \notin B_{\frac{\epsilon_0}{2}}(T_0(x_0)).$$

Since $\delta_n \downarrow 0$, we have $\{y_n\}_{n=1}^\infty \subset T_\eta(x_0)$. By the compactness of $\overline{B}_\eta(T(x_0))$, there exists a subsequence of $\{y_n\}$ which converges to some $y_0 \in \overline{B}_\eta(T(x_0))$. Without loss of generality, we may assume that $y_n \to y_0$. Thus, for any $\epsilon > 0$, we have $\delta_n < \frac{\epsilon}{2}$ and $d(y_n, y_0) < \frac{\epsilon}{2}$ for all sufficiently large n. Since $y_n \in T_{\delta_n}(x_0)$, taking $f_n \in C_{\delta_n}(T)$ such that $f_n(x_0) = y_n$, we have

$$f_n(x) \in B_{\delta_n}(T(x)) \subset B_{\frac{\epsilon}{2}}(T(x)),\ \forall\ x \in X.$$

Now, we define a multifunction $F : X \longrightarrow 2^Y$ by

$$F(x) = \begin{cases} \overline{B}_{\frac{\epsilon}{2}}(f_n(x)) & , \text{if } x \neq x_0, \\ \{y_0\} & , \text{if } x = x_0. \end{cases}$$

Therefore, for each sufficiently large n, it is easy to check that F is $l.s.c.$, and each $F(x)$ is a closed set. According to Theorem 1.1, F admits a continuous selection g_n. By Lemma 1.5, the above information yields

$$g_n(x) \in F(x) \subset \overline{B}_{\frac{\epsilon}{2}}(f_n(x)) \subset \overline{B}_{\frac{\epsilon}{2}}(B_{\frac{\epsilon}{2}}(T(x))) \subset B_\epsilon(T(x)),\ \forall\ x \in X,$$

i.e., $g_n \in C_\epsilon(T)$, and hence,

$$y_0 = g_n(x_0) \in T_\epsilon(x_0),\ \forall\ \epsilon > 0.$$

It follows that $y_0 \in T_0(x_0)$. But for all sufficiently large n, $y_n \in (B_{\frac{\epsilon_0}{2}}(T_0(x_0)))^C$ implies $y_0 \in (B_{\frac{\epsilon_0}{2}}(T_0(x_0)))^C$, which contradicts with $y_0 \in T_0(x_0)$. Moreover, if $T_0(x) = \emptyset$ for some $x \in X$, then $T_\delta(x) \subset B_{\frac{\epsilon}{2}}(T_0(x)) = \emptyset$. This leads a contradiction, by Proposition 2.2. Thus, $T_0(x) \neq \emptyset$ for all $x \in X$. Hence, the proof is complete. □

Proposition 2.4. *Let Y be a complete metric space, and $T : X \longrightarrow 2^Y$ be an ECP multifunction. If for each $x \in X$, there exists some $\eta := \eta(x) > 0$ such that $\overline{B}_\eta(T(x))$ is compact, then for any $\epsilon > 0$,*

$$T_{\frac{\sigma}{2}}(x) \subset B_\epsilon(T_0(x)),\ \forall\ x \in X,$$

where $T_0 : X \longrightarrow 2^Y$ is defined in Proposition 2.3, and $\sigma := \sigma(\frac{\epsilon}{2})$ is taken as in the definition of ECP.

Proof. For any fixed $x \in X$ and $\epsilon > 0$, by using the ECP of T, there exist a corresponding positive number $\sigma := \sigma(\frac{\epsilon}{2})$ and a neighborhood N_x of x, such that

(1) $d(f(z), f(x)) < \frac{\sigma}{2},\ \forall\ f \in C_{\frac{\sigma}{2}}(T),\ \forall\ z \in N_x$;
(2) $d(y_1, y_2) < \frac{\epsilon}{2},\ \forall\ y_1, y_2 \in \bigcap_{z\in N_x} B_\sigma(T(z))$.

We first claim that $d(y_1, y_2) < \frac{\epsilon}{2}$ for all $y_1, y_2 \in T_{\frac{\sigma}{2}}(x)$. By the definition of $T_{\frac{\sigma}{2}}(x)$, there exist $f_1, f_2 \in C_{\frac{\sigma}{2}}(T)$ such that $f_1(x) = y_1$ and $f_2(x) = y_2$. It follows that for $i = 1, 2$, we have

$$y_i = f_i(x) \in B_{\frac{\sigma}{2}}(f_i(z)) \subset B_{\frac{\sigma}{2}}(B_{\frac{\sigma}{2}}(T(z))) \subset B_\sigma(T(z)),\ \forall\ z \in N_x.$$

Thus, $y_1, y_2 \in \bigcap_{z\in N_x} B_\sigma(T(z))$, and hence, $d(y_1, y_2) < \frac{\epsilon}{2}$.

Now, applying Proposition 2.3, we have some $\delta := \delta(x, \epsilon) > 0$ such that $T_\delta(x) \subset B_{\frac{\epsilon}{2}}(T_0(x))$. If $\delta \geq \frac{\sigma}{2}$, it is clear that

$$T_{\frac{\sigma}{2}}(x) \subset T_\delta(x) \subset B_{\frac{\epsilon}{2}}(T_0(x)) \subset B_\epsilon(T_0(x)).$$

If $\delta < \frac{\sigma}{2}$, we are able to show that $T_{\frac{\sigma}{2}}(x) \subset \overline{B}_{\frac{\epsilon}{2}}(T_\delta(x))$. Assume not. Then there exists $y_1 \in T_{\frac{\sigma}{2}}(x)$ but $y_1 \notin \overline{B}_{\frac{\epsilon}{2}}(T_\delta(x))$, i.e., $d(y_1, y_2) > \frac{\epsilon}{2}$ for all $y_2 \in T_\delta(x)$. This is impossible because $y_2 \in T_\delta(x) \subset T_{\frac{\sigma}{2}}(x)$. Thus, by Lemma 1.5, we still have

$$T_{\frac{\sigma}{2}}(x) \subset \overline{B}_{\frac{\epsilon}{2}}(T_\delta(x)) \subset \overline{B}_{\frac{\epsilon}{2}}(B_{\frac{\epsilon}{2}}(T_0(x))) \subset B_\epsilon(T_0(x)).$$

□

Proposition 2.5. *Under the same condition of Proposition 2.4, if $T(x)$ is closed for all $x \in X$, then $T_0 : X \longrightarrow 2^Y$ is a l.s.c. multifunction such that $T_0(x) \subset T(x)$ for all $x \in X$.*

Proof. First, we notice that $T_0(x) \neq \emptyset,\ \forall\ x \in X$, by Proposition 2.3. Next, for any $\epsilon > 0$, by Proposition 2.4, we have $\sigma := \sigma(\frac{\epsilon}{4}) > 0$ such that

$$T_{\frac{\sigma}{2}}(x) \subset B_{\frac{\epsilon}{2}}(T_0(x)),\ \forall\ x \in X.$$

Now, for any $y \in T_0(x)$, we have $y \in T_{\frac{\sigma}{2}}(x)$. By Proposition 2.2, $T_{\frac{\sigma}{2}}$ is $l.s.c.$. Thus, for each $x \in X$, there exists a neighborhood V_x of x, such that $y \in B_{\frac{\epsilon}{2}}(T_{\frac{\sigma}{2}}(z))$ for all $z \in V_x$, by Lemma 1.4. It follows that

$$y \in B_{\frac{\epsilon}{2}}(B_{\frac{\epsilon}{2}}(T_0(z))) \subset B_\epsilon(T_0(z)),\ \forall\ z \in V_x.$$

This shows that T_0 is also *l.s.c.*. Further, since $T(x)$ is closed for all $x \in X$, we have

$$T_0(x) := \bigcap_{\epsilon>0} T_\epsilon(x) \subset \bigcap_{\epsilon>0} B_\epsilon(T(x)) = \overline{T(x)} = T(x), \ \forall\, x \in X.$$

Therefore, the proof is complete. □

According to the above propositions, we are now in a position to establish our main selection theorem, which extends Theorem 1.1 to *a.l.s.c.* multifunctions, and improves Corollary 1.3, instead of the one-point extension property.

Theorem 2.6. *Let X be paracompact, with $\dim X = 0$, Y a complete metric space, and $T : X \longrightarrow 2^Y$ be a multifunction. If there is an a.l.s.c. ECP multifunction $S : X \longrightarrow 2^Y$ satisfying*

(1) *each $S(x)$ is closed, and $S(x) \subset T(x)$, $\forall\, x \in X$,*
(2) *for each $x \in X$, $\overline{B}_\eta(S(x))$ is compact for some $\eta > 0$,*

then T admits a continuous selection.

Proof. Let $S_\epsilon(x) := \{f(x) \mid f \in C_\epsilon(S)\}$ for each $\epsilon > 0$. Then by a parallel argument of Proposition 2.2, the multifunction $S_\epsilon : X \longrightarrow 2^Y$ is *l.s.c.*. Further, by Proposition 2.5, the multifunction S_0, defined by $S_0(x) := \bigcap_{\epsilon>0} S_\epsilon(x)$, $\forall\, x \in X$, is also *l.s.c.*. In addition, $S_0(x) \neq \emptyset$ and $S_0(x) \subset S(x)$ for all $x \in X$. It follows from Lemma 1.6 that $\overline{S_0}$ is also a *l.s.c.* multifunction having nonempty closed images. By Michael'selection theorem (see Theorem 1.1), $\overline{S_0}$ admits a continuous selection f, and hence,

$$f(x) \in \overline{S_0(x)} \subset \overline{S(x)} = S(x) \subset T(x), \ \forall\, x \in X.$$

This says that f is also a continuous selection for T. □

Corollary 2.7. *Let X be paracompact, with $\dim X = 0$, and Y be a complete metric space. If $T : X \longrightarrow 2^Y$ is an a.l.s.c. ECP multifunction satisfying*

(1) *$T(x)$ is closed for all $x \in X$,*
(2) *for each $x \in X$, $\overline{B}_\eta(T(x))$ is compact for some $\eta > 0$,*

then T admits a continuous selection.

Corollary 2.8. *Let X be paracompact, with $\dim X = 0$, and Y be a complete metric space. If $T : X \longrightarrow 2^Y$ is an a.l.s.c. ECP multifunction such that $T(x)$ is compact for all $x \in X$, then T admits a continuous selection.*

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