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SEPARATELY HOLOMORPHIC FUNCTIONS ON COMPACT SETS

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Abstract. The main aim of this paper is to give a characterization for $\mathcal{H}(Z)$ to have the property (DN), where Z is a Stein space, and to give conditions for a compact set K in a locally irreducible Stein space not to be pluripolar in every irreducible branch of all neighbourhoods of K.

1. Introduction

Let K be a compact set in a complex space X and Z a complex space. For a function $f : K \times Z \longrightarrow \mathbb{C}$ we put

$$f_x(z) = f(x,z) \quad \text{for } z \in Z,$$
$$f^z(x) = f(x,z) \quad \text{for } x \in K.$$

The function f is called *separately holomorphic* if $f_x : Z \longrightarrow \mathbb{C}$ and $f^z : K \longrightarrow \mathbb{C}$ are holomorphic for all $x \in K$ and $z \in Z$, respectively. Here a function on K is said to be *holomorphic* if it can be extended holomorphically to a neighbourhood of K in X.

The aim of the present note is to find some conditions on K and Z for which every separately holomorphic function on $K \times Z$ is holomorphic.

For the formulation of results we need the following notions.

1.1. The properties (DN) and (LB^∞)

Let E be a Frechet space with a fundamental system of semi-norms $\{\| \, . \, \|_k\}_{k=1}^\infty$. For each subset $B \subset E$, define the generalized semi-norm $\| \, . \, \|_B^* : E' \longrightarrow [0, +\infty]$, where E' is the dual space of E, by

$$\|u\|_B^* = \sup\{|u(x)| : x \in B\}.$$

We will write $\| \, . \, \|_k^*$ for $\| \, . \, \|_{U_k}^*$, where $U_k = \{x \in E : \|x\|_k < 1\}$.

Using this notion, we say that E has the property

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(DN) if $\exists p\ \forall q,\ d>0\ \exists k,\ C>0: \|\ .\ \|_q^{1+d} \le C \|\ .\ \|_k \|\ .\ \|_p^d$,

(LB^∞) if $\forall \rho_N \uparrow \infty\ \forall p\ \exists q\ \forall k\ \exists n_k, C>0\ \forall u \in E'\ \exists k \le n \le n_k$:

$$\| u \|_q^{*1+\rho_N} \le C \| u \|_n^* \| u \|_p^{*\rho_N}.$$

The above properties were introduced and investigated by Vogt (see, for example, [8], [9]). In these cases we will write $E \in (DN)$ and $E \in (LB^\infty)$, respectively.

1.2. Plurisubharmonic functions

Given a complex space X and a function $\varphi : X \longrightarrow [-\infty, +\infty)$. We say that φ is *plurisubharmonic* on X if φ is upper-semicontinuous and plurisubharmonic on the regular locus $R(X)$ of X. Note that such a function is called by Zeriahi [11] weakly plurisubharmonic. A subset Y of X, for which there exists a plurisubharmonic function φ on X such that $\varphi\big|_Y = -\infty$ and $\varphi \not\equiv -\infty$ on every irreducible branch of X, is called the *pluripolar set* in X.

1.3. The results

In this note we prove the following two theorems:

Theorem A. *Let Z be a Stein space. The following conditions are equivalent:*

(i) *The space $\mathcal{H}(Z)$ of holomorphic functions on Z equipped with the compact-open topology has the property (DN).*

(ii) *Every separately holomorphic function on $K \times Z$, where K is a compact set in a locally irreducible Stein space X which is not pluripolar in every irreducible branch of all neighbourhoods of K, can be extended holomorphically to a neighbourhood $W \times Z$ of $K \times Z$ in $X \times Z$.*

Theorem B. *Let K be a compact set in a locally irreducible Stein space X. Then the following conditions are equivalent:*

(i) *K is not pluripolar in every irreducible branch of all neighbourhoods of K.*

(ii) *$[\mathcal{H}(K)]' \in (LB^\infty)$, where $\mathcal{H}(K)$ denotes the space of holomorphic functions on K equipped with the inductive topology.*

(iii) *Every separately holomorphic function on $K \times Z$, where Z is a Stein space, $\mathcal{H}(Z) \in (DN)$ and K is unique, can be extended holomorphically to a neighbourhood $W \times Z$ of $K \times Z$ in $X \times Z$.*

The proofs of Theorems A and B are given in Section 2 and Section 3, respectively.

2. Proof of Theorem A

For the proof of the theorem we need the following three lemmas.

Lemma 2.1 [3]. *Let $\theta : Y \longrightarrow Z$ be a finite proper holomorphic surjection between Stein spaces. Then $\mathcal{H}(Z) \in (DN)$ if and only if $\mathcal{H}(Y) \in (DN)$.*

Lemma 2.2 [2]. *Let Z be a locally irreducible Stein space. Then $\mathcal{H}(Z) \in (DN)$ if and only if every plurisubharmonic function on Z, which is bounded from above, is constant.*

Lemma 2.3. *Let Z be a Stein space. Then $\mathcal{H}(Z) \in (DN)$ if and only if $\mathcal{H}(Z \setminus H) \in (DN)$ for all hypersurfaces $H \subset Z$ containing the singular locus $S(Z)$ of Z.*

Proof. Since $\mathcal{H}(Z)$ is contained in $\mathcal{H}(Z \setminus H)$ as a subspace for every hypersurface H in Z (see [5]), the sufficiency is obvious.

Conversely, assume that $\mathcal{H}(Z) \in (DN)$ and H is a hypersurface in Z containing $S(Z)$. Since $Z \setminus H$ is a Stein manifold, it suffices to show that every plurisubharmonic function φ on $Z \setminus H$, which is bounded from above, is constant. Consider the normalization $\theta : \widetilde{Z} \longrightarrow Z$ of Z. Since $\varphi\theta$ is plurisubharmonic on $\widetilde{Z} \setminus \theta^{-1}(H)$ and locally bounded on $\widetilde{Z}$, by the normality of $\widetilde{Z}$ it follows that $\varphi\theta$ can be considered as a plurisubharmonic function on $\widetilde{Z}$. By Lemma 2.1 $\mathcal{H}(\widetilde{Z}) \in (DN)$. Lemma 2.2 then yields that $\varphi\theta$ and hence φ is constant.

Now we are able to prove Theorem A.

(i) $\Longrightarrow$ (ii). Given a separately holomorphic function $f : K \times Z \longrightarrow \mathbb{C}$, where K is a compact set in a locally irreducible Stein space X which is not pluripolar in every irreducible branch of all neighbourhoods of K.

Let $\{W_n\}$ be a neighbourhood basis of K and T, H are hypersurfaces in X, Z, respectively, such that $S(X) \subset T$ and $S(Z) \subset H$. For each $n \geq 1$ put

$$Z_n = \{z \in Z \setminus H : f^z \in \mathcal{H}(W_n), \|f^z\|_{W_n} \leq n\},$$

where $\|f^z\|_{W_n}$ denotes the sup-norm of f^z on W_n. From the separate holomorphicity of f we have

$$Z \setminus H = \bigcup_{n \geq 1} Z_n.$$

On the other hand, by the Montelness of $\mathcal{H}(W_n)$, it follows that Z_n are closed in $Z \setminus H$. The Baire Theorem yields that there exists n_0 such that $\operatorname{Int} Z_{n_0} \neq \emptyset$. Note that $\operatorname{Int} Z_{n_0}$ meets every irreducible branch of Z. Writing $K \cap (W_{n_0} \setminus T)$ as a countable union of compact sets in $W_{n_0} \setminus T$, we can find a compact set $E \subset K \cap (W_{n_0} \setminus T)$ which is not pluripolar in every connected component of $W_{n_0} \setminus T$. Now we can consider f as a separately holomorphic function on $(E \times Z \setminus H) \cup (W_{n_0} \setminus T \times \operatorname{Int} Z_{n_0})$ (in the sense of Siciak [7]). From the relation $\mathcal{H}(Z \setminus H) \in (DN)$ and from the non-pluripolarrity of E in every connected component of $W_{n_0} \setminus T$ it follows by Zaharjuta [10] that f is extended to a holomorphic function $\widehat{f}$ on $W_{n_0} \cap (X \setminus T) \times (Z \setminus H)$.

Consider the holomorphic function from W into $\mathcal{H}(Z \setminus H)$ given by

$$x \longmapsto \widehat{f}_x, \quad x \in W,$$

where $W = W_{n_0} \cap (X \setminus T)$. Since E is not pluripolar and $\mathcal{H}(Z)$ is contained in $\mathcal{H}(Z \setminus H)$ as a subspace with

$$\left\{ \widehat{f}_x \ : x \in E \right\} \subset \mathcal{H}(Z),$$

$\widehat{f}$ can be considered as a holomorphic function on $W_{n_0} \cap (X \setminus T) \times Z$.

Similarly, using the holomorphic function

$$z \longmapsto \widehat{f}^z \in \mathcal{H}(W_{n_0} \cap (X \setminus T)), \quad z \in Z,$$

we can consider $\widehat{f}$ as a holomorphic function on $W_{n_0} \times Z$.

(ii) $\Longrightarrow$ (i). By Vogt [9], it suffices to check that every continuous linear map T from $\mathcal{H}(\triangle)$ into $\mathcal{H}(Z)$ is compact, where $\triangle = \left\{ \lambda \in \mathbb{C} : |\lambda| < 1 \right\}$. Since $[\mathcal{H}(\triangle)]' \cong \mathcal{H}(\overline{\triangle})$, the map T induces a function $f : \overline{\triangle} \times \mathbb{C} \longrightarrow \mathbb{C}$ by

$$f(\lambda, z) = (T^* \delta_z)(\lambda) \text{ for } (\lambda, z) \in \overline{\triangle} \times \mathbb{C},$$

where

$$\delta_z(\varphi) = \varphi(z) \text{ for } \varphi \in \mathcal{H}(\mathbb{C}).$$

Obviously, f is separately holomorphic. By the hypothesis, f is holomorphically extended to a holomorphic function $\widehat{f}$ on a neighbourhood $W \times \mathbb{C}$ of $\overline{\triangle} \times \mathbb{C}$. This implies that T^* maps continuously $[\mathcal{H}(\mathbb{C})]'$ into $\mathcal{H}^\infty(V)$, where V is a relatively compact neighbourhood of $\overline{\triangle}$ in W and $\mathcal{H}^\infty(V)$ is the Banach space of bounded holomorphic functions on V. Hence T is compact.

3. Proof of Theorem B

We need the following

Lemma 3.1. *Let K be a compact set in a complex space X such that $[\mathcal{H}(K)]' \in (LB^\infty)$. Then K is unique.*

Proof. Given $f \in \mathcal{H}(K)$ with $f\big|_K = 0$. Let $\{U_k\}$ be a neighbourhood basis of K. For each $k \geq 1$, put

$$\varepsilon_k = \|f\|_{U_k} = \sup\{|f(x)| : x \in U_k\}.$$

Then $\varepsilon_k \downarrow 0$. By applying (LB^∞) to $\rho_N = \sqrt{-\log \varepsilon_N} \uparrow +\infty$ we have $f \in \mathcal{H}^\infty(U_p)$ for $p \geq 1$ and

$$\exists q\ \forall N\ \exists \widetilde{N} \geq N, \quad C_N > 0\ \forall n\ \exists N \leq k_n \leq \widetilde{N} :$$

$$\|f^n\|_q^{1+\rho_{k_n}} \leq C_N \|f^n\|_{k_n}\ \|f\|_p^{\rho_{k_n}}.$$

This yields

$$\|f\|_q^{1+\rho_{k_n}} \leq C_N^{1/n} \|f\|_{k_n}\ \|f\|_p^{\rho_{k_n}}.$$

Choose $N \leq k \leq \widetilde{N}$ such that

$$\#\{n : k_n = k\} = \infty.$$

Then

$$\big\|f\big\|_q \leq \big\|f\big\|_k^{\frac{1}{1+\rho_k}}\ \big\|f\big\|_p^{\frac{\rho_k}{1+\rho_k}} = \big(\varepsilon_k\big)^{\frac{1}{1+\sqrt{-\log\varepsilon_k}}}\ \big(\varepsilon_p\big)^{\frac{\sqrt{-\log\varepsilon_k}}{1+\sqrt{-\log\varepsilon_k}}} \longrightarrow 0$$

as $k \longrightarrow \infty$. Hence $f = 0$ on V_q.

Now we prove Theorem B.

(i) $\Longrightarrow$ (iii) by Theorem A.

(iii) $\Longrightarrow$ (i). Assume that there exists an irreducible branch Z of a neighbourhood U of K such that $E = K \cap Z$ is pluripolar. Since Z is a connected component of U, it follows that E satisfies the hypothesis of (iii).

Choose a plurisubharmonic function φ on X for which $\varphi\big|_E = -\infty$. Let W be a neighbourhood of E in X for which there exists a finite proper holomorphic map θ from W onto the unit polydisc $\triangle^n$, $n = \dim X$. Consider the plurisubharmonic function $\widetilde{\varphi}$ on $\triangle^n \setminus S(\theta)$ given by

$$\widetilde{\varphi}(z) = \sum_{\theta(x)=z} \varphi(x), \quad z \in \triangle^n \setminus S(\theta),$$

where $S(\theta)$ denotes the branch locus of θ. Since θ is proper, $\widetilde{\varphi}$ is bounded from above on $\triangle^n$. Hence

$$\widehat{\varphi}(z) = \lim\sup\big\{\widetilde{\varphi}(z') : z' \longrightarrow z,\ z' \in \triangle^n \setminus S(\theta)\big\}$$

is a plurisubharmonic extension of $\widetilde{\varphi}$. This function is also equal to $-\infty$ on $\theta(E)$.

Indeed, let $z \in \theta(E)$. Write $\theta^{-1}(z) = \{x^1, \dots, x^q, x^{q+1}, \dots, x^p\}$ with

$$\varphi(x^j) = -\infty \text{ for } 1 \le j \le q,$$

and

$$\varphi(x^j) \ne -\infty \text{ for } q+1 \le j \le p.$$

Given $M > 0$. For each $j = 1, \dots, p$ take a neighbourhood U_j of x^j such that

$$\varphi(x) < -M \quad \text{for } x \in U_j\,, \quad j = 1, \dots, q,$$

and

$$\varphi(x) < \varphi(x^j) + 1 \quad \text{for } x \in U_j\,, \quad q+1 \le j \le p.$$

We may assume that U_j are disjoint. Since θ is proper, there exists a neighbourhood V of z such that

$$\theta^{-1}(V) \ \subset \bigcup_{j=1}^{p} U_j.$$

It follows that for $z' \in V \setminus S(\theta)$ we have

$$\begin{aligned}\widehat{\varphi}(z') = \widetilde{\varphi}(z') &= \sum \Big\{\varphi(x') : \theta(x') = z'\,, \quad x' \in \bigcup_{1 \le j \le q} U_j\Big\} \\ &\quad + \sum \Big\{\varphi(x') : \theta(x') = z'\,, \quad x' \in \bigcup_{q+1 \le j \le p} U_j\Big\} \\ &\le -M + (p-q-1)\max\big(\varphi(x^j) + 1\big)\,.\end{aligned}$$

Hence $\widehat{\varphi}\big|_{\theta(E)} = -\infty$.

Consider the Hartogs domain $\Omega_{\widehat{\varphi}}$ in $\triangle^n \times \mathbb{C}$ given by

$$\Omega_{\widehat{\varphi}} \ = \Big\{(z, \lambda) \in \triangle^n \times \mathbb{C} : |\lambda| < e^{-\widehat{\varphi}(z)}\Big\}.$$

Since $\Omega_{\widehat{\varphi}}$ is a pseudoconvex domain, there exists $f \in \mathcal{H}(\Omega_{\widehat{\varphi}})$ such that $\Omega_{\widehat{\varphi}}$ is the domain of existence of f (see [4]). Write the Hartogs expansion of f on $\Omega_{\widehat{\varphi}}$,

$$f(z,\lambda) = \sum_{n\geq 0} f_n(z)\lambda^n,$$

where

$$f_n(z) = \frac{1}{2\pi i} \int\limits_{|\lambda| = e^{-\delta\widehat{\varphi}(z)}} \frac{f(z,\lambda)}{\lambda^{n+1}} d\lambda \quad (\delta > 1).$$

Since the sequence $\left\{\frac{1}{n}\log|f_n(z)|\right\}$ is locally bounded, for each $m \geq 1$ we can define

$$\begin{aligned}\Psi_m(z) &= \sup\left\{\frac{1}{n}\log|f_n(z)| : n \geq m\right\} \quad \text{for } z \in \triangle^n,\\ \Psi_m^*(z) &= \limsup_{z' \longrightarrow z} \Psi_m(z').\end{aligned}$$

By Bedford-Taylor [1] Ψ_m^* is plusubharmonic on $\triangle^n$ and the set $\Big\{\Psi_m < \Psi_m^*\Big\}$ is pluripolar. Let

$$\widehat{\Psi} = \lim_{m \longrightarrow \infty} \Psi_m^*.$$

It suffices to show that $\widehat{\Psi}$ is not identically equal to $-\infty$ on every non-empty open set in $\triangle^n$.

Assume by contrary that $\widehat{\Psi} \equiv -\infty$ on a non-empty open set $U \subset \triangle^n$. Then the Hartogs theorem [4] implies that the series $\sum\limits_{n\geq 0} f_n(z)\lambda^n$ converges to a holomorphic function g on $U \times \mathbb{C}$. This yields $U \times \mathbb{C} \subset \Omega_{\widehat{\varphi}}$ and hence $\widehat{\varphi}\big|_U = -\infty$. It follows that $\widehat{\Psi}$ is plurisubharmonic on $\triangle^n$ and the set $\Big\{\Psi < \widehat{\Psi}\Big\}$ is pluripolar, where $\Psi = \lim\limits_{m \longrightarrow \infty} \Psi_m$.

Consider the function:

$$g(x,\lambda) = f(\theta(x),\lambda) \text{ for } x \in E \text{ and } \lambda \in \mathbb{C}.$$

By the hypothesis we can find a holomorphic function $\widehat{g}$ on a neighbourhood $W \times \mathbb{C}$ of $E \times \mathbb{C}$ such that $\widehat{g}\big|_{E\times\mathbb{C}} = g$. Choose a neighbourhood V of E in W such that

$$V \times \triangle \subset \Big\{(x,\lambda) : (\theta x, \lambda) \in \Omega_{\widehat{\varphi}}\Big\}.$$

Consider the Hartogs expansion of $\widehat{g}$ on V $\times\triangle$:

$$\widehat{g}(x,\lambda)=\sum_{n\geq 0}\widehat{g}_n(x)\lambda^n.$$

Then

$$\widehat{g}_n\big|_E=f_n\theta\big|_E \quad \text{for } n\geq 0.$$

Hence, shrinking V if necessary, we have $\widehat{g}_n\big|_V=f_n\theta\big|_V$ for $n\geq 0$. This yields

$$\begin{aligned}-\infty&=\limsup_n \frac{1}{n}\log|\widehat{g}_n(x)|=\limsup_n \frac{1}{n}\log|f_n\theta(x)|\\&=\Psi(\theta(x))=\widehat{\Psi}(\theta(x)),\end{aligned}$$

for $x\in V\setminus\theta^{-1}(\{\Psi<\widehat{\Psi}\})$, which is impossible.

(i) $\Longrightarrow$ (ii). To prove $[\mathcal{H}(K)]'\in(LB^\infty)$, by Vogt [9], it suffices to show that every continuous linear map $T:[\mathcal{H}(K)]'\longrightarrow\mathcal{H}(\mathbb{C})$ is compact. Define the function

$$f_T(x,\lambda)=T(\delta_x)(\lambda)\quad \text{for } x\in K,\ \lambda\in\mathbb{C}.$$

This function is separately holomorphic. By Theorem A there is a holomorphic extension $\widehat{f}_T$ of f_T to a neighbourhood $V\times\mathbb{C}$ of $K\times\mathbb{C}$. Since

$$\mathcal{H}(V,\mathcal{H}(\mathbb{C}))\cong\mathcal{H}(V)\hat{\otimes}_\pi\mathcal{H}(\mathbb{C})\cong\mathcal{L}\left([\mathcal{H}(V)]',\mathcal{H}(\mathbb{C})\right),$$

the form

$$S(\delta_z)(\lambda)=\widehat{f}_T(z)(\lambda)\quad \text{for } z\in V,\ \lambda\in\mathbb{C},$$

defines a continuous linear map from $[\mathcal{H}(V)]'$ into $\mathcal{H}(\mathbb{C})$. By the uniqueness of K, from the relations

$$\begin{aligned}T\Big(\sum_j\lambda_j\delta_{z_j}\Big)&=\sum_j\lambda_jT(\delta_{z_j})=\sum_j\lambda_jf_T(z_j)\\&=\sum_j\lambda_jS(\delta_{z_j})=S\Big(\sum_j\lambda_j\delta_{z_j}\Big).\end{aligned}$$

it follows that $T=S$. Hence T is compact.

(ii) $\Longrightarrow$ (i). We show that $E=K\cap Z$ is not pluripolar, where Z is an irreducible branch of a neighbourhood U of K.

Assume by contrary that E is pluripolar. We use the same notations in the proof of (iii) $\Longrightarrow$ (i). Consider the linear map $T : [\mathcal{H}(\mathbb{C})]' \longrightarrow \mathcal{H}(E)$ given by

$$(T\mu)(x) = \langle g_x, \mu\rangle \quad \text{for } \mu \in [\mathcal{H}(\mathbb{C})]' \quad \text{and } x \in E.$$

The definition is correct. Indeed, given $\mu \in [\mathcal{H}(\mathbb{C})]'$, choose $C > 0$ and $r > 0$ such that

$$|\ \langle \sigma, \mu\rangle\ | \leq C\|\sigma\|_{r\triangle} \quad \text{for } \sigma \in \mathcal{H}(\mathbb{C}).$$

Let V be a neighbourhood of E in X such that $V \times r\triangle \subset (\theta \times id)^{-1}\Omega_{\widehat{\varphi}}$. Write

$$g(x,\lambda) = \sum_{n\geq 0} g_n(x)\lambda^n \quad \text{for } x \in V,\ |\lambda| < r\,.$$

Since

$$|\langle \lambda^n, \mu\rangle| \leq Cr^n \quad \text{for } n \geq 0,$$

it follows that the series $\sum\limits_{n\geq 0} g_n(x)\langle \lambda^n, \mu\rangle$ converges uniformly to $\langle g_x, \mu\rangle$ on a relatively compact neighbourhood of E in V. This means that $T(\mu) \in \mathcal{H}(E)$.

Further, since U is locally irreducible, Z is a connected component of U and hence $[\mathcal{H}(E)]' \in (LB^\infty)$. By Lemma 3.1 E is an unique set. This yields that T has a closed graph. By the open mapping Grothendieck Theorem in [6], T is continuous. By Vogt [9], T continuously maps $[\mathcal{H}(\mathbb{C})]'$ into $\mathcal{H}(V)$ for some neighbourhood V of E. Then the form

$$\widehat{g}(x,\lambda) = T(\delta_\lambda)(x)\,, \quad x \in V\,,\ \lambda \in \mathbb{C}$$

defines a holomorphic extension of g to $V \times \mathbb{C}$. Similarly as in (iii) $\Longrightarrow$ (i) we get a contradiction.

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