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ON THE PROBLEM OF AIR POLLUTION

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Abstract. One of the first steps in studying problems of mathematical modelling of environment pollution is the consideration of the correctness of the posed problems. We shall investigate the existence and uniqueness of the solutions under some general assumptions concerning the right-hand member of equation describing an air pollution process, and we give some exact solutions for nonstationary problems.

1. Introduction

Let D be a cylindrical region in the space R^3 with sufficiently smooth boundary. Denote by Ω the set $D\times(0,T)=\{(x,t):x=(x_1,x_2,x_3)\in D,$ $0<t<T<\infty\}$.

The process of pollutant transport and diffusion in the atmosphere is described by the following equations [4]

$$LF=\frac{\partial F}{\partial t}-\operatorname{div}\lambda\bigtriangledown F+\operatorname{div}\vec{V}F+\sigma F=f \text{ in } \Omega \tag{1}$$

$$\operatorname{div}\vec{V}=0 \tag{2}$$

where $F=F(x,t)$ is the concentration of pollutant, $\vec{V}=(u,v,w)$ is the wind velocity, $f=f(x,t)$ is the power of the source, $\lambda=\lambda(x)$ is the diffusion coefficient and $\sigma=\sigma(x)$ is the rate of chemical decay transformation, and $\lambda(x)$ and $\sigma(x)$ are continuous functions in D, $0\le\lambda(x)\le\lambda_0$; $0\le\sigma(x)\le\sigma_0$; λ_0, σ_0 being constants.

The mixed problem of an air pollution process [4] consists of equations (1)-(2) with the initial condition

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$$F = I(x) \quad \text{for } t = 0, \quad x \in D, \tag{3}$$

and boundary conditions

$$\begin{aligned}
&F = F_c \quad \text{on the lateral surface } \partial D_c \quad \text{if } V_n = \vec{V}.\vec{n} < 0,\\
&\vec{n} \quad \text{is the outer normal to } \partial D,\\
&\frac{\partial F}{\partial n} = 0 \quad \text{on } \partial D_c \quad \text{if } V_n \equiv V_n^+ \geq 0,\\
&\frac{\partial F}{\partial x_3} + \beta F = 0 \quad \text{on the bedding surface } \partial D_0 \quad (x_3 = 0),\\
&\text{and} \quad V_n = 0, \quad \beta = \beta(x) \in C(\partial D_0),\\
&0 \leq \beta(x) \leq \beta_0, \quad \beta_0 \text{ is a constant},\\
&\frac{\partial F}{\partial x_3} = 0 \quad \text{on the upper base } \partial D_H \quad \text{and } V_n = 0,
\end{aligned} \tag{4}$$

where $\partial D_c \cup \partial D_0 \cup \partial D_H = \partial D,\ t \in (0, T)$.

We shall consider the correctness of the posed problem with some more general boundary conditions for the equation (1) in an open cylinder $\Omega = D \times (0, T)$, $D \subset R^\ell$, (ℓ is a natural number):

$$\begin{aligned}
&F = F_1 \quad \text{on } \partial D_1 \quad \text{and} \quad V_n < 0,\\
&\frac{\partial F}{\partial n} = 0 \quad \text{on } \partial D_2 \quad \text{and } V_n \equiv V_n^+ \geq 0,\\
&\frac{\partial F}{\partial n} + \beta\ F = 0 \quad \text{on } \partial D_3 \quad \text{and} \quad V_n = V_n^+ \geq 0,
\end{aligned} \tag{4'}$$

where $\partial D_1 \cup \partial D_2 \cup \partial D_3 = \partial D, \quad t \in (0, T)$.

2. Proof of uniqueness

We shall deal with the generalized solution in Ω of the problem (1)-(3), (4'). Since here we consider the uniqueness for solution of a boundary value problem of mathematical physics, the concentration $F(x, t)$ always satisfies the condition $F(x, y) \geq 0 \quad \forall (x, y) \in \overline{\Omega}$. It is well known that for the first boundary condition, we may always assume without loss of generality, that

$$F = 0 \quad \text{on } \partial D_1 \times (0, T). \tag{4''}$$

Let $\breve{H}^1(\Omega)$ be the space of all functions $F(x,t)$ which satisfy the condition (4") and which belong to $L^2(\Omega)$ together with all their generalized derivatives DF. The scalar product in $\breve{H}^1(\Omega)$ is defined by the same way as in $H^1(\Omega)$. Let

$$\breve{H}^1(D) = \Big\{ h : h \in H^1(D),\ h = 0 \text{ on } \partial D_1 \Big\}.$$

Following [3] we denote by $L^2(0,T;\breve{H}^1(D))$ the space of all functions $F(x,t)$: $t \to F(x,t)$ of $(0,T) \to \breve{H}^1(D)$ such that

$$\int_0^T \big\| F(x,t) \big\|^2_{\breve{H}^1(D)} dt < \infty.$$

Let f be a given function in $L^2(\Omega)$ and $I(x) \in L^2(D)$.

Definition. A function $F \in L^2(0,T,\breve{H}^1(D))$ is said to be a generalized solution in Ω of the problem (1)-(3), (4') if it satisfies the following equality

$$\begin{aligned} &\int_\Omega \big[-F\psi_t + \lambda \nabla F . \nabla \psi + \psi \operatorname{div} \vec{V} F + \sigma F \psi \big] dxdt \\ &+ \int_{\partial\Omega_3} \lambda \beta F \psi ds dt = \int_{\partial\Omega^0} I \psi dx + \int_\Omega f \psi dx dt, \end{aligned} \tag{5}$$

for any $\psi(x,t) \in H^1(\Omega)$, $\dfrac{\partial \psi}{\partial t} \in L^2(\Omega)$ with

$$\psi\big|_{\partial\Omega^T \cup \partial\Omega_1} = 0, \tag{6}$$

where

$$\begin{aligned} &\partial\Omega^0 = \big\{(x,t) : x \in D,\ t = 0\big\}, \\ &\partial\Omega^T = \big\{(x,t) : x \in D,\ t = T\big\}, \\ &\partial\Omega_i = \big\{(x,t) : x \in \partial D_i,\ t \in (0,T)\big\}, \quad i = 1,2,3, \\ &\partial\Omega_1 \cup \partial\Omega_2 \cup \partial\Omega_3 = \partial\Omega = \big\{(x,t) : x \in \partial D,\ 0 < t < T\big\}. \end{aligned}$$

It is easy to verify that the classical solution of the continuously differentiable class for Problem (1)-(3),(4') is its generalized solution.

Theorem 1. *The initial boundary problem* (1)-(3), (4') *can't have more than one generalized solution in* $L^2(0,T;\breve{H}^1(D))$.

Proof. Let F satisfy the homogeneous integral equality corresponding to (5) with $f=0$ and $I=0$. We shall show that $F=0$ in $L^2(0,T,\breve{H}^1(D))$. Consider the function ψ in Ω :

$$\psi(x,t)=\int_t^T F(x,\tau)d\tau.$$

Then

$$\psi\big|_{\partial\Omega^T}=0,\quad \psi\big|_{\partial\Omega_1}=\int_t^T F\big|_{\partial\Omega_1}d\tau=0,\quad \psi_t=-F.$$

Putting the function ψ into (5) we obtain

$$\int_\Omega\Big[F^2+\lambda\nabla F\cdot\int_t^T\nabla F d\tau-\sigma\psi\psi_t+\mathrm{div}\vec{V}F\int_t^T F d\tau\Big]dxdt \tag{7}$$
$$+\int_{\partial\Omega_3}\lambda\beta F\int_t^T F(x(s),\tau)d\tau dsdt=0.$$

One has

$$\int_\Omega\lambda(x)\nabla F(x,t)\cdot\int_t^T\nabla F(x,\tau)d\tau dxdt=\frac{1}{2}\int_D\lambda\Big|\int_0^T\nabla F dt\Big|^2dx\geq 0,$$
$$\int_\Omega\sigma\psi(x,t)\psi_t(x,t)dxdt=-\frac{1}{2}\int_{\partial\Omega^0}\sigma\psi^2dx\leq 0,$$
$$\int_{\partial\Omega_3}\lambda\beta F(x(s),t)\int_t^T F(x(s),\tau)d\tau dsdt=\frac{1}{2}\int_{\partial\Omega_3}\lambda\beta\Big[\int_0^T F(x(s),t)dt\Big]^2ds\geq 0.$$

Applying the Gauss formula and taking into account that $F(x,t)\geq 0$ $\forall(x,t)\in\overline{\Omega}$, we get from assumption (4')

$$\int_D F\mathrm{div}F^*\vec{V}dx=\int_{\partial D_2\cup\partial D_3}F^*V_n^+ds\geq 0, \tag{8}$$

where $F^* = F(x,t)F(x,\tau)$, $0 \le t \le \tau$, $0 \le \tau \le T$. Therefore,

$$\int_{\Omega} \operatorname{div} F\vec{V} \int_{t}^{T} F(x,\tau)d\tau dx dt \ge 0. \tag{8'}$$

From (7) it follows that

$$\int_{\Omega} F^2(x,t)dxdt \le 0.$$

Hence $F = 0$, which proves the theorem.

As an immediate consequence we obtain

Corollary 1. *The mixed problem* (1)-(3), (4') *has at most one classical solution in the class of continuously differentiable functions.*

3. Proof of existence

Like for the space $H^1(D)$ [5] it can be showed that $\breve{H}^1(D)$ is separable. Therefore, there exists a "basis" $G_1, G_2, \ldots, G_m, \ldots$ in $\breve{H}^1(D)$ and the (finite) linear combinations of the elements G_k, $k = 1, \ldots, m, \ldots$ are dense in $\breve{H}^1(D)$. We will find an "approximate solution" of the problem (1)-(3), (4') in the form

$$F_m(x,t) = \sum_{k=1}^{m} g_{km}(t)G_k(x), \tag{9}$$

where the $g_{km}(t) \in H^1(0,T)$ are defined by the following system:

$$\begin{aligned} &\int_D \Big\{ \Big[\frac{\partial F_m}{\partial t} + \operatorname{div} \vec{V} F_m + \sigma F_m \Big] G_\ell + \lambda \nabla F_m \cdot \nabla G_\ell \Big\} dx \\ &\quad + \int_{\partial D_3} \lambda \beta F_m G_\ell ds = \int_D f G_\ell dx, \quad \ell = 1, 2, \ldots, m \end{aligned} \tag{10}$$

$$F_m(x,0) = I^m(x). \tag{11}$$

Here $I^m(x)$ is the orthogonal projection of $I(x)$ on the space V_m (generated by the system $G_1, G_2, \dots, G_m$):

$$I^m(x) = \sum_{k=1}^{m} I_k^m G_k(x),$$

and

$$I^m(x) \to I(x) \quad \text{as } m \to \infty. \tag{11'}$$

The system (10), (11) is a linear system of m differential equations with constant coefficients :

$$K_m \frac{dg_m}{dt} + L_m g_m(t) = f_m(t)$$
$$g_m(0) = \{I_k^m\}, \quad k = 1, 2, \dots, m,$$

where the matrix coefficients K_m, L_m are defined by (10). Since

$$\det K_m = \det ||(G_k, G_\ell)|| \neq 0; \quad k, \ell = 1, 2, \dots, m,$$

the problem (10), (11) has a unique solution $g_m(t) = \{g_{km}(t)\}$.

We now show that as $m \to \infty$, $F_m \to F$, where F is the generalized solution of problem(1)-(3), (4'). To do this, we multiply (10) by $g_{\ell m}$, sum up over ℓ and integrate over $(0, T)$. Then we obtain

$$\begin{aligned} &\int\limits_{\Omega} \Big[\frac{\partial}{\partial t}\Big(\frac{1}{2}F_m^2\Big) + \lambda|\nabla F_m|^2 + F_m \text{ div } \vec{V} F_m + \sigma F_m^2\Big] dxdt \\ &\quad + \int\limits_{\partial\Omega_3} \lambda\beta F_m^2 dsdt = \int\limits_{\Omega} f.F_m dxdt. \end{aligned} \tag{12}$$

In view of (11) we get

$$\frac{1}{2}\int\limits_{\Omega} \frac{\partial}{\partial t} F_m^2 dxdt = \frac{1}{2}\int\limits_{\partial\Omega^T} F_m^2 dx - \frac{1}{2}\big\|I^m(x)\big\|^2_{L^2(D)}. \tag{13}$$

By analogy with (8),

$$\int\limits_{\Omega} F_m \; div \; \vec{V} F_m dxdt = \frac{1}{2} \int\limits_{\partial\Omega_2 \cup \partial\Omega_3} F_m^2 V_m^+ dsdt \geq 0. \tag{14}$$

Let $\breve{H}^1(D)$ be endowed with the norm

$$\|\hat{F}\|_{\breve{H}^1(D)} = \Big(\int_D (\lambda|\nabla\hat{F}|^2 + \sigma\hat{F}^2)dx\Big)^{1/2}. \tag{15}$$

Summing up (12)-(15) we get

$$\begin{aligned}\|F_m(x,t)\|^2_{L^2(0,T;\breve{H}^1(D))} &\le \frac{1}{2}\|I^m(x)\|^2_{L^2(D)} + \int_\Omega |f|\;|F_m|dxdt\\ &\le \frac{1}{2}\|I^m(x)\|^2_{L^2(D)} + \|F_m\|_{L^2(\Omega)}\cdot\|f\|_{L^2(\Omega)}\\ &\le \frac{1}{2}\|I^m(x)\|^2_{L^2(D)} + \frac{1}{2}\|F_m\|^2_{L^2(0,T;\breve{H}^1(D))}\\ &\quad + \frac{1}{2}\|f\|^2_{L^2(\Omega)}.\end{aligned}$$

Hence

$$\|F_m(x,t)\|^2_{L^2(0,T;\breve{H}^1(D))} \le \|I^m(x)\|^2_{L^2(D)} + \|f\|^2_{L^2(\Omega)} = C. \tag{16}$$

Thus, the set $\{F_m\}$ is bounded in $L^2(0,T;\breve{H}^1(D))$ and it is possible to extract a subsequence (denote by $\{F_m\}$ too) such that

$$F_m \to K(x,t) \quad \text{weakly in} \quad L^2(0,T;\breve{H}^1(D)). \tag{17}$$

We shall show that $K = F$ is the desired solution. To this end, we first multiply both sides of (10) by $\varphi(t)$, where

$$\varphi(t),\ \varphi'(t) \in L^2[0,T], \quad \varphi(T) = 0. \tag{18}$$

Let ℓ be arbitrarily fixed. Setting $\psi_\ell(x,t) = \varphi(t)G_\ell(x)$ and integrating over $(0,T)$, by virtue of (11) we get, for $m \ge \ell$,

$$\begin{aligned}&\int_\Omega \Big[-F_m\frac{\partial\psi_\ell}{\partial t} + (\text{div}\,\vec{V}F_m + \sigma F_m)\psi_\ell + \lambda\nabla F_m.\nabla\psi_\ell\Big]dxdt\\ &\quad + \int_{\partial\Omega_3} \lambda\beta F_m\psi_\ell dsdt = \int_{\partial\Omega^0} I^m\psi_\ell dx + \int_\Omega f\psi_\ell dxdt.\end{aligned} \tag{19}$$

Further, since ℓ is arbitrary and the finite linear combinations of G_ℓ are dense in $L^2(0,T;\breve{H}^1(D))$, letting $m \to \infty$ in (19) we obtain that K is the generalized solution of the problem (1)-(3), (4'). So we obtain the following result

Theorem 2. *The mixed problem* (1)-(3), (4') *admits a generalized solution in* $L^2(0,T;\breve{H}^1(D))$. *This solution satisfies the estimate* (16).

Remark. When $\beta_0 \leq \beta < 0$ Theorem 2 can be proved similarly.

It is easy to verify that a generalized solution of the problem (1)-(3), (4') in the class of continuously differentiable functions (in the classical sense) is a classical one.

4. Exact solution

Since the right-hand side $f(x,t)$ (the power of source) of the differential equation describing an air pollution process is often a density distribution of masses concentrated at distinct points or on the surfaces, we next consider the mixed problem (1)-(3), (4') in the space of distributions.

Note that we have proved the uniqueness and existence of the solution in the space of mixed distribution for differential operators with variable coefficients corresponding to the generalized solution of the mixed problem (1)-(3), (4') [2].

Let $\Omega \subset \overline{R}^1_+ \times R^n = \{(x,t) : x \in R^n, t \in [0,\infty]\}$ be an open set, ∂D a bounded piecewise smooth two-sided surface. We now wish to find a solution in $\mathcal{D}'(\overline{R}^1_+ \times R^n)$ of our mixed problem.

Assuming that $\vec{V} = \overrightarrow{\text{const}}$, $\lambda, \sigma = \text{const} > 0$, we obtain (1) in the following form

$$PF = \frac{\partial F}{\partial t} - \lambda \triangle F + \vec{V}.\nabla F + \sigma F = f \text{ in } \Omega. \tag{1'}$$

We rewrite the condition on ∂D_3 in the form

$$\lim_{(x,t)\to(x_s,t_s)} \frac{\partial F}{\partial n} = -\beta F_0(x_s,t_s), (y,\tau) \in \partial\Omega_3, \tag{4"'}$$

where $F_0(y,\tau)$ is a given continuous functions on $\partial\Omega_3$.

In accordance with the general theory [6] the differential equation corresponding to (1') for the mixed problem (1'), (3) and (4') (with (4") and

(4"') in $\mathcal{D}'(\overline{R}^1_+ \times R^n)$) has the form

$$PF̂ = \hat{f} + \overset{\frown}{I(x)} \times \delta(t) + \hat{F}_3\delta_{\partial\Omega} = K \tag{20}$$

where $\hat{F}$ is the extended distribution of F by zero onto $\Omega^- = R^1 \times R^\ell \setminus \overline{\Omega}$,

$$\hat{f} = \begin{cases} f, & 0 < t < T, \quad x \in \mathcal{D}, \\ 0, & t < 0, \quad x \in R^n, \end{cases} \tag{21}$$

provided f is a finite (with respect x) distribution on $\mathcal{D}$, $\delta(t)$ is the Dirac distribution, $\hat{F}_3\delta_{\partial\Omega}$ is the generalized simple layer on $\partial\Omega$ with surface density $\hat{F}_3$ uniquely defined by the function $\hat{F}_0\big|_{\partial\Omega_3}$, $\hat{F}_0$ is the extended function of F_0 by zero onto $\partial\Omega_1 \cup \partial\Omega_2$, by the surface $\partial\Omega$ and the operator $P(.)$, and $\overset{\frown}{I}(x)$ is the extended distribution of $I(x)$ by zero onto D^- (the complement of D with respect to R^n).

We consider the fundamental solution $E(x,t)$ of (20) in $\mathcal{D}'(R^1_+ \times R^n)$:

$$PE = \delta(x,t). \tag{22}$$

Applying the Fourier transform $\mathcal{F}_x$ to (22) we obtain

$$\frac{\partial \mathcal{F}_x[E](\xi)}{\partial t} + \big[- i\vec{V}.\vec{\xi} + \sigma + \lambda|\vec{\xi}|^2\big]\mathcal{F}_x(E) = 1(\xi) \times \delta(t).$$

The solution in $S'(R^1_+ \times R^n) \subset \mathcal{D}'(R^1_+ \times R^n)$ of the last equation is

$$\mathcal{F}_x[E](\xi,t) = \theta(t)_e^{[i\vec{V}.\vec{\xi} - \sigma - \lambda|\vec{\xi}|^2]t}.$$

Therefore,

$$E(x,t) = \mathcal{F}_\xi^{-1}[\mathcal{F}_x(E)] = \frac{\theta(t)}{(4\lambda\pi t)^{n/2}} \exp\Big\{ - [\sigma t + \frac{|x - \vec{V}t|^2}{4\lambda t}]\Big\}, \tag{23}$$

where $\theta(t)$ is the Heaviside unit function. Thus, the unique solution in $\mathcal{D}'(\overline{R}^1_+ \times R^n)$ of the mixed problem (20), (3), (4') is expressed by the form of a "dispersion potential"

$$\hat{F} = K * E = \hat{f} * E + [\overset{\frown}{I}(x) \times \delta(t)] * E + \hat{F}_3\delta_{\partial\Omega} \overset{s}{*} E. \tag{24}$$

We next consider the summands of the dispersion potential. Let $\hat{f} \in L^2(\Omega)$ and $\hat{f}$ satisfy the following estimate in each strip $0 \leq t \leq T$, $x \in R^n$:

$$|f(x,t)| \leq C_{T,\epsilon}(f)e^{\epsilon|x|^2} \tag{25}$$

for an arbitrary $\epsilon > 0$, where the quantity $C_{T,\epsilon}$ does not decrease with respect to T. Then, in view of (21) one has

(26)

$$f_1 = \hat{f} * E =$$

$$\int_0^T \int_{R^n} \frac{\hat{f}(\xi,\tau)}{(4\lambda\pi(t-\tau)^{n/2}} \exp\Big\{ - \Big[\sigma(t-\tau) + \frac{|(x-\xi) - \vec{V}(t-\tau)|^2}{4\lambda(t-\tau)}\Big]\Big\} d\xi d\tau.$$

It follows that the potential f_1 satisfies the following estimate

$$|f_1(x,t)| \leq \frac{C_{t,\epsilon}(f)e^{2\epsilon|x|^2}}{(1-16\epsilon\lambda t)^{n/2}} t.e^{4\epsilon|\vec{V}|^2t^2} \quad \text{for} \quad 0 < t < \frac{1}{16\epsilon\lambda} \cdot \tag{27}$$

Thus, for arbitrary $A > 0$,

$$f_1(x,t) \xrightarrow[t\to+0]{|x|<A} 0. \tag{28}$$

We now consider the surface dispersive potential f_2:

$$f_2(x,t) = [\widehat{I}(x) \times \delta(t)] * E = \frac{\theta(t)e^{-\sigma t}}{(4\lambda\pi t)^{n/2}} \int_{R^n} \widehat{I}(y) e^{-\frac{|x-y-\vec{V}t|^2}{4\lambda t}} dy \tag{29}$$

where the density $I(x) \in L^2(D)$ is a finite function in D. Using the equality

$$\frac{1}{\pi^{n/2}} \int_{R^n} e^{-|\xi|^2} d\xi = 1$$

we get

$$\lim_{t\to 0} f_2(x,t) = I(x). \tag{30}$$

The third summand f_3 in (20) is the generalized potential of simple layer on $\partial\Omega$:

$$f_3(x,t) = \hat{F}\delta_{\partial\Omega} \overset{s}{*} E = \int_0^T \int_{\partial D} \hat{F}_3(x - x_s - y, t - t_s - \tau)E(y,\tau) ds d\tau, \tag{31}$$

where $(x,t)\in\Omega, (x_s,t_s)$ is an arbitrary fixed point on $\partial\Omega_3$ and

$$\hat{F}_3=\begin{cases} +\frac{\beta}{C_{n,D}}F_0(x_s,t_s)\hat{F}_0\big|_{\partial D_3}, & x\in\partial D_3, \quad t\in(0,T)\\ 0, \quad x\in\partial D_1\cup\partial D_2, & t\in(0,T)\\ 0, \quad x\in\overline{D}, & t\leq 0\end{cases}$$

(32)

$$C_{n,D}=\int\limits_0^T\int_{\partial D_3}F_0(-y,-\tau)\frac{\partial E(y,\tau)}{\partial n_y}ds_yd\tau$$

which depends on $\hat{F}_0\big|_{\partial D_3}$ and the dimension of D. One has

$$\frac{\partial f_3}{\partial n}=-\frac{\beta}{C_{n,D}}F_0(x_s,t_s)\int\limits_0^T\int\limits_{\partial D_3}F_0(x-x_s-y,t-t_s-\tau)\frac{\partial E(y,\tau)}{\partial n_y}ds_yd\tau$$

where $F_0(y,\tau)=\widehat{F_0}|_{\partial D_3}$. Hence, we get

$$\lim_{(x,t)\to(x_s,t_s)}\frac{\partial f_3}{\partial n}=-\beta F_0(x_s,t_s). \tag{33}$$

Finally, it is easy to verify that $P_{c\ell}E=0$ for $(x,t)\in\Omega$, where $P_{c\ell}$ is the differential operator P with classical derivatives. Hence, by virtue of (21), (28), (30), (32) and (33), we find that the dispersive potential is the classical solution of the mixed problem (1’), (3), (4’).

We have thus proved the following result

Theorem 3. *The unique solution in* $\mathcal{D}'(\overline{R}^1_+\times R^n)$ *of the generalized problem for air pollution* (20), (3), (4’) *is expressed by the form* (24) *of a dispersive potential. The classical solution in the class of continuously differentiable functions for the problem* (1’),(3), (4’) *is given as a sum of three potentials*

$$\begin{aligned}F=&\int\limits_0^t\int\limits_{R^n}\frac{f(\xi,\tau)}{[4\lambda\pi(t-\tau)]^{n/2}}\exp\Big\{-\sigma(t-\tau)+\frac{|(x-\xi)-\vec{V}(t-\tau)|^2}{4\lambda(t-\tau)}\Big\}d\xi d\tau\\ &+\frac{\theta(t)e^{-\sigma t}}{(4\lambda\pi t)^{n/2}}\int\limits_{R^n}\widehat{I}(y)\exp\Big\{-\frac{|x-y-\vec{V}t|^2}{4\lambda t}\Big\}dy\\ &+\int\limits_0^T\int\limits_{\partial D}\hat{F}_3(y,\tau)E(x-x_s-y,t-t_s-\tau)ds_yd\tau, \quad (x,t)\in\Omega.\end{aligned}$$

where the function E is defined by (23) and $\hat{F}_3(x,t)$ by (32).

Remark. Finally, in the special case $\beta = 0$ we give in [1] a numerical solution of the problem (1)-(3), (4).

References

1. Nguyen Cong Dieu, *Programs for solving the three dimensional air pollution problem*, Institute of Informatic Technology, NCNST 1993.
2. Hoang Dinh Dung, *On the quasimodules and their application*, National Conference of Mathematicians IV, Hanoi 1990.
3. J. L. Lion, *Problems aux limites non homogenes et application*, v.1, Dunod, Paris 1968.
4. G. I. Marchuk, *Mathematical modelling in the environment problems*, Nauka, Moscow 1982 (in Russian).
5. V. P. Mikhailov, *Equations aux derivees partielles*, Mir, Moscou 1980.
6. V. S. Vladimirov, *Generalized functions in mathematical physics*, Mir, Moscow 1979.

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