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FIRING SEQUENCES AND PROCESSES OF PETRI NETS

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Abstract. We show how an equivalence relation on the set of firing sequences of a general net can be defined so that equivalent sequences have the same set of processes. For a special class of nets, we establish a characterization of equivalence classes on firing sequences which define the same partial orders on transition occurrences as those defined by the processes. With every n-safe net we associate a 1-safe net which has, in a certain sense, the same set of processes as the initial net. For some classes of nets we obtain some kind of regular properties for the set of processes.

1. Introduction

There are two widely used ways for describing the behaviour of a net by its firing sequences and by its processes. Relationships between these two notions have been investigated in [2, 6, 7, 11].

For 1-safe Petri nets, Mazurkiewicz's traces are equivalent to the processes presented in [8, 2]. From the set of firing sequences and an equivalence relation derived from the structure of a 1-safe net it is easy to determine all of its processes. But this can not be done for general nets as shown in [2]. There E. Best introduced an equivalence relation based on the dynamic behaviour of nets and analyzed the relationship between firing sequences and processes. In general it is hard to find a nice relation between firing sequences and processes. In this paper we try to give answers to the following questions:

- How can we define an equivalence relation on the set of transitions of a general net so that the elements of an equivalence class have the same corresponding sets of processes?

- For which nets can we construct the set of processes from the set of firing sequences and the knowledge on the structure of the net?

Some earlier terminology and results of [1, 2, 3, 4, 6, 7, 9, 10] will be recalled in Section 2. In Section 3 we define a semi-commutation on transitions which

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is based only on the structure of net. Then we show that the corresponding equivalence relation on the set of firing sequences meets the requirement of the first question. Furthermore, we prove that when the parallel occurrences of the same transition are not allowed, there exists a reduced set of firing sequences such that two of its elements are equivalent if and only if they define the same set of processes.

For the general case, the sets of processes corresponding to different equivalence classes can overlap. Hence we cannot get a nice relationship between firing sequences and processes. We will introduce a class of nets in which there is a global observation for every process of net. A net of this class is called a globally observable net. We show that for such nets the set of equivalence classes generated by the above mentioned reduced set of firing sequences and the set of processes define the same partial orders of transition occurrences.

In Section 4 we consider the behaviour of n-safe nets. We present a method to construct a 1-safe net from a given n-safe net so that the 1-safe net has, in a certain sense, the same behaviour as the n-safe net. From this and the fact that the set of firing sequences of a 1-safe net is regular, we deduce a regular characterization of the set of processes of an n-safe net. This correspondence is bijective if in the n-safe net no transition occurs concurrently with itself. For the globally observable nets the regularity of the set of firing sequences implies the regularity of the set of processes in the sense that there exists a regular trace language which is equivalent to the set of processes as labelled partial orders.

2. Basic definitions and notions

In this section we recall some terminology and results.

DEFINITION 2.1.

(i) A triple (S,T,F) is a net iff S and T are disjoint sets, $F \subseteq (S\times T)\cup(T\times S)$ and $T \subset dom(F)\cup cod(F)$. The relation F is interpreted as a function: $F(x,y) = 1 \Leftrightarrow (x,y) \in F$.

(ii) $\Sigma = (S,T,F,M_0)$ is a system net (or Petri net) iff (S,T,F) is a net and M_0: $S \to \mathbf{N}$ is a marking, where $\mathbf{N}$ is the set of positive integers.

(iii) A net $N = (B,E,F)$ is an occurrence net iff $\forall s \in S$: $|{}^\bullet s|\leq 1 \ \wedge \ |s^\bullet|\leq 1$ and F^+ (the transitive closure of F) is acyclic.

Here we use the notations:

$${}^\bullet x = \{y \in S\cup T \mid (y,x) \in F\} \text{ and } x^\bullet = \{y \in S\cup T \mid (x,y) \in F\}.$$

DEFINITION 2.2. Let $N = (B, E, F)$ be an occurrence net.

(i) We define two sets **li, co** $\subseteq (B \cup E) \times (B \cup E)$ by

$$(x, y) \in \mathbf{li} \Leftrightarrow (x < y \vee y < x \vee x = y).$$
$$(x, y) \in \mathbf{co} \Leftrightarrow ((x, y) \notin \mathbf{li}) \vee x = y).$$

(ii) $\mathbf{l} \subseteq B \cup E$ is a **li-set** iff $\forall x, y \in \mathbf{l}$: $(x, y) \in \mathbf{li}$.

(iii) $\mathbf{c} \subseteq B \cup E$ is a **co-set** iff $\forall x, y \in \mathbf{c}$: $(x, y) \in \mathbf{co}$.

(iv) The interval between two **co-sets** c_1, c_2 is defined by:

$$[c_1, c_2] = \{z \in B \cup E \mid \exists x \in c_1\ \exists y \in c_2: x \leq z \leq y\}.$$

(v) A net N is discrete with respect to **c** iff

$\forall x \in B \cup E\ \exists n \in \mathbf{N}$: $\forall$ **li-sets l**: $|[\mathbf{c}, x] \cap \mathbf{l}\,| \leq n \wedge |[x, \mathbf{c}] \cap \mathbf{l}\,| \leq n$.

DEFINITION 2.3. Let $\Sigma = (S, T, F)$ be a net, M a marking and $t, t' \in T$.

(i) M enables t iff $\forall s \in S$: $F(s, t) \leq M(s)$.

(ii) M' is produced from M by the firing of t iff M enables t and $\forall s \in S$: $M'(s) = M(s) - F(s, t) + F(t, s)$. In that case, we write $M[t\!> M'$.

(iii) M enables concurrently $\{t_i \mid 1 \leq i \leq k\}$ iff $\forall s \in S$: $\sum_{i=1}^{k} F(s, t_i) \leq M(s)$.

DEFINITION 2.4. Let $\Sigma = (S, T, F, M_0)$ be a system net.

(i) $\sigma = M_0 t_1 M_1 \ldots t_i M_i \ldots$ is a firing sequence of Σ iff $\forall i \geq 1$: $M_{i-1}[t_i\!> M_i$. Sometime one may use the reduced form of σ: $\sigma = t_1 t_2 \ldots t_i \ldots$ and $M_0[t_1 \ldots t_i\!> M_i$. The marking M_i is called a reachable marking from M_0.

(ii) The set of all finite and infinite firing sequences of Σ is denoted by $\mathbf{F}(\Sigma)$.

(iii) $[M_0\!>$ will denote the set of all reachable markings of the net Σ from the initial marking M_0.

(iv) Σ is a 1-safe iff $\forall M \in [M_0\!>, \forall s \in S$: $M(s) \leq 1$.

(v) Σ is an n-safe iff $\exists n \in \mathbf{N}$ such that $\forall M \in [M_0\!>, \forall s \in S$: $M(s) \leq n$.

(vi) Σ is a self-concurrency free iff $\forall t \in T$ and $\forall M \in [M_0\!>$: $\{t, t\}$ are not enabled concurrently at M.

DEFINITION 2.5. Let $\Sigma = (S, T, F, M_0)$ be a system net, $N = (B, E, F')$ an occurrence net and p a mapping: $B \cup E \rightarrow S \cup T$. The pair (N, p) is a process of Σ iff

(i) $p(B) \subseteq S, p(E) \subseteq T$.

(ii) $Min(N)$ is a *B-cut* of N, i.e., a maximal **co-set** consisting of elements of B.

(iii) N is discrete with respect to $Min(N)$.

(iv) $\forall e \in E, s \in S$: $F(s, p(e)) = |p^{-1}(s) \cap {}^\bullet e| \wedge F(p(e), s) = |e^\bullet \cap p^{-1}(s)|$.

(v) $\forall s \in S$: $M_0(s) = |p^{-1}(s) \cap Min(N)|$.

From the practical point of view we only consider the countable system nets that are degree-finite (i.e., $\forall x \in B \cup E$: $x^\bullet$ and ${}^\bullet x$ are finite sets) and have finite markings. For a process $\pi = (B, E, F, p)$ of a system net, we call $O_\pi = (E, \leq_\pi, p)$ the labelled partial ordering derived from π, where $\leq_\pi = F^*|_E$. The basic relationship between processes and firing sequences presented in [3] is based on the following construction.

CONSTRUCTION 2.6. Let $\Sigma = (S, T, F, M_0)$ be a system net and $\sigma = M_0 t_1 M_1 \ldots$ be a firing sequence of Σ. A set $\pi(\sigma)$ of processes is associated to σ as follows. We construct labelled occurrence nets $(N_i, p_i) = (B_i, E_i, F_i, p_i)$, where $i \in \mathbf{N}$ and p_i: $B_i \cup E_i \rightarrow S \cup T$, by a recursive procedure:

Define $E_0 = F_0 = \emptyset$ and B_0 containing, for each $s \in S$, $M_0(s)$ distinct conditions b with $p_0(b) = s$. Suppose (N_i, p_i) has been already constructed. For each $s \in {}^\bullet t_{i+1}$ we choose a condition $b(s) \in Max(N_i) \cap p_i^{-1}(s)$. Then we add a new event e with $p_{i+1}(e) = t_{i+1}$. Also for each $s \in t_{i+1}^\bullet$ we add a new condition $b' = b'(s)$ with $p_{i+1}(b') = s$ such that $(e, b') \in F_{i+1}$ for all $s \in t_{i+1}^\bullet$. For $x, y \in B_i \cup E_i$ we define $p_{i+1}(x) = p_i(x), (x, y) \in F_{i+1} \Leftrightarrow (x, y) \in F_i$.

For $\sigma = M_0 t_1 \ldots t_n M_n$ the procedure stops at $i = n$, and we put $\pi = (N, p) \in \pi(\sigma)$ with $N = N_n$ and $p = p_n$. If σ is infinite, we put $\pi = (\cup B_i,\ \cup E_i,\ \cup F_i,\ \cup p_i) \in \pi(\sigma)$.

The following theorem is taken from [4].

THEOREM 2.7. *Let Σ be a system net that is degree-finite with a finite initial marking. Then*

(i) For each firing sequence σ of Σ, $\pi(\sigma)$ is a set of processes of Σ.

(ii) For each process π of Σ there exists a firing sequence σ such that $\pi \in \pi(\sigma)$.

DEFINITION 2.8. Let $\pi_i = (B_i, E_i, F_i, p_i), i = 1, 2$ be processes of a system net Σ. We define $\pi_1 \approx \pi_2$ iff there is a bijection β: $E_1 \rightarrow E_2$ such that $\forall e, e_1, e_2 \in E_1$: $((p_1(e) = p_2(\beta(e)) \wedge (e_1 <_1 e_2 \Leftrightarrow \beta(e_1) <_2 \beta(e_2)))$, where $<_i = F_i^+$.

For a system net Σ, let $\mathbf{P}(\Sigma)$ denote the set of processes of Σ.

THEOREM 2.9. *Let Σ be a system net, $\pi = (B, E, F, p)$ be a process of Σ and let $\sigma = t_1 t_2 \ldots$ be a sequence of transitions of Σ. Then σ is a firing sequence of Σ and $\pi \in \pi(\sigma)$ if and only if there exists a bijection β: $E \rightarrow \{1, 2, \ldots\}$ such that*

$$\forall e, e_1, e_2 \in E\colon ((p(e) = t_\beta(e)) \wedge (e_1 <_\pi e_2 \Rightarrow \beta(e_1) < \beta(e_2)))$$

PROOF. It immediately follows from 2.10 and the proof of 2.10 in [2].

DEFINITION 2.10. Let π be a process of a system net Σ. We denote the *Lin-set* of π by

$$Lin(\pi) = \{\sigma \mid \sigma \text{ is a firing sequence of } \Sigma \text{ and } \pi \in \pi(\sigma)\}.$$

A semi-commutative system $SC = <A, R>$ is a semi-Thue system, where A is a finite set alphabet and R is a set of rules of the form $ab \rightarrow ba$ with $a, b \in A$ and $a \neq b$. If all rules in R are symmetrical, then we say $<A, R>$ is a commutative system. For a semi-commutative system $SC = <A, R>$ let R_S denote the set of symmetrical rules of R, i.e.

$$R_S = \{ab \rightarrow ba \mid ab \rightarrow ba \in R \wedge ba \rightarrow ab \in R\}.$$

We say $<A, R_S>$ is the commutative system derived from $<A, R>$. We write $x \rightarrow_R y$ for $x, y \in A^*$ if $x = x_1 ab x_2, y = x_1 ba x_2$ and $ab \rightarrow ba \in R$. The reflexive and transitive closure $\rightarrow_R$ is denoted by $\rightarrow_R^*$. A semi-trace generated by a string x will be defined as a set of all words derived from x by rules of R and denoted by $[x>_R$, i.e., $[x>_R = \{y \in A^* \mid x \rightarrow_R^* y\}$.

We define an equivalence relation associated with a commutative system $<A, R_S>$: $x \equiv y$ iff $x \rightarrow_{R_S}^* y$. Equivalence class is denoted by $[x]_{R_S}$ and will be called a trace.

Let us fix some semi-commutative system $<A, R>$. Unless there is no confusion, we omit the subscripts R, R_S for semi-traces and traces. A semi-commutative monoid over $<A, R>$ is a triple $(M, \circ, \{\epsilon\})$, where $M = \{[x> \mid x \in A^*\}$, $[x> \circ [y> = [x.y>$. We denote a free partial commutative monoid over $<A, R_S>$ by $(M_S, \circ, \{\epsilon\})$, where $M_S = \{[x] \mid x \in A^*\}$ and $[x] \circ [y] = [x.y]$.

The following lemma is obvious from the above mentioned notations (see [6,10]).

LEMMA 2.11. $[u> = [v> \Longleftrightarrow [u] = [v]$.

DEFINITION 2.12. Let $w = a_1 a_2 \ldots a_n \in A^*$. Set

$$O(w) = \{(a, k) \mid a \in alph(w), k \in \mathbf{N}, 1 \leq k \leq |w|_a\}$$

We define an ordering $\leq_w$ on $O(w)$ as follows: $(a, k) \leq_0 (a', m)$ iff $\exists ua'\colon w \in ua'T^*$ with $|ua'|_{a'} = m$ and $\exists va\colon ua' \in vaT^*$ with $|va|_a = k$ and $aa' \rightarrow a'a \notin R$. For $\leq_\sigma = (\leq_0)^*$, the labelled partial ordering derived from σ is denoted by $O_\sigma = (O(\sigma), \leq_\sigma, lb)$, where $lb((a, n)) = a$, for all $a \in A$, and all $n \in \mathbf{N}$.

Let us consider the following orderings on M and M_S between semitraces and traces of the same length:

$$[\alpha> \leq [\beta> \quad \text{iff} \quad [\alpha> \subseteq [\beta>, \text{ i.e. } \beta \rightarrow^* \alpha.$$
$$[\alpha] \leq [\beta] \quad \text{iff} \quad [\alpha> \leq [\beta>, \text{ i.e. } \beta \rightarrow^* \alpha.$$

LEMMA 2.13. (see [6]). *Let $w, w' \in A^*$. Then*

$$[w'> \leq [w> \Longleftrightarrow (O(w) = O(w') \wedge \leq_w \subseteq \leq_{w'}).$$

3. Semi-commutation and equivalence relation on firing sequences

As shown by E. Best [2], from the "independence relation" [2, 3] on transitions one cannot in general derived an equivalence relation on firing sequences such that each equivalence class corresponds exactly to one process.

In this section we introduce a semi-commutation on transitions of a net. Then we give another notion of the equivalence on firing sequences in order to find a finer relationship between processes and firing sequences. We show that the "independence relation" which is derived from a semi-commutation based on the structure of nets can generate an equivalence relation on firing sequences in which two equivalent sequences have the same set of processes. When the parallel occurrences of the same transition are not allowed and there is a global observation for every process, the set of equivalence classes generated by those global observations can replace the set of processes.

Firstly we want to extend the definition of finite semi-traces and traces to infinite semi-traces and traces. Let $SC = <A, R>$ be a semi-commutative system and $A^\omega = A^* \cup A^\infty$.

DEFINITION 3.1. Let $x, y \in A^\omega$. Then $x \rightarrow^\omega y$ iff $\forall$ prefix u of y $\exists v$ prefix of x and $w \in A^*$ such that: $v \rightarrow^* uw$. We define and denote semi-traces and traces corresponding by

$$[x> = \{y \in A^\omega \mid x \rightarrow^\omega y\},\ [x] = \{y \in A^\omega \mid x \rightarrow^\omega y \wedge y \rightarrow^\omega x\}.$$

and derive the labelled partial ordering $(O(x), \leq_x, lb)$ from $x \in A^\omega$ as follows:

$$O(x) = \cup O(u) \quad \text{and} \quad \leq_x = \cup \leq_u \text{ for all prefixes } u \text{ of } x.$$

From Definition 3.1 and Lemma 2.13 it immediately follows

LEMMA 3.2. *Let $x, y \in A^\omega$. Then*

(i) $x \rightarrow^\omega y$ iff $O(y) \subseteq O(x)$ and $\leq_x \subseteq \leq_y$,

(ii) $[x> = [y>$ iff $[x] = [y]$.

DEFINITION 3.3. Let $\Sigma = (S, T, F, M_0)$ be a system net. The semi-commutative system derived from Σ is $SC(\Sigma) = <T, R>$, where

$$R = \{tt' \rightarrow t't \mid t^{\bullet} \cap {}^{\bullet}t = \emptyset \wedge t \neq t'\}.$$
$$R_S = \{tt' \rightarrow t't \mid t^{\bullet} \cap {}^{\bullet}t' = t'^{\bullet} \cap {}^{\bullet}t = \emptyset \wedge t \neq t'\}.$$

For a system net Σ we always take the labelled partial ordering $(O_\sigma, \leq_\sigma, lb)$ from a firing sequence σ, and we deal with semi-traces and traces in this derived semi-commutative system $SC(\Sigma)$.

The next theorem shows that our definition meets the above mentioned requirements.

THEOREM 3.4. *Let Σ be a system net and σ_1, σ_2 be firing sequences of Σ. If $\sigma_1 \equiv \sigma_2$ then $\pi(\sigma_1) = \pi(\sigma_2)$.*

PROOF. See [7].

The following example shows that the converse of 3.4 is not true.

EXAMPLE 3.5. Let Σ be a system net given in Fig. 1.

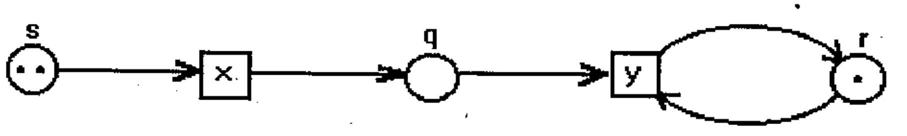

Fig. 1

$xxy \not\equiv xyx$, but $\pi(xxy) = \pi(xyx)$ contains unique process shown in Fig. 2.

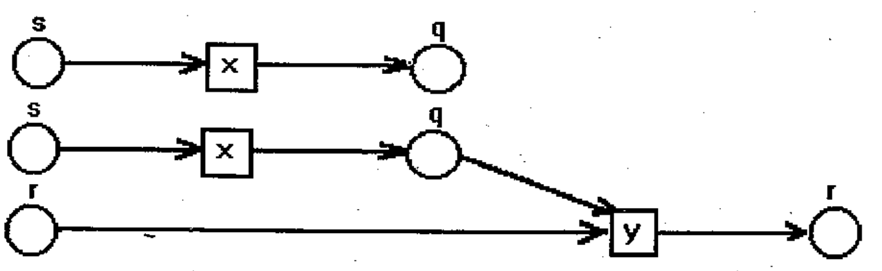

Fig. 2

DEFINITION 3.6. Let $\Sigma = (S, T, F)$ be a net and let $\sigma \in T^\omega$. A word σ is strict with the transition $t \in alph(\sigma)$ iff for all prefixes γ of σ which has the form $\gamma = \beta t$, there exists an injective mapping τ: $Max(\beta) \cap {}^{\bullet}t \rightarrow S$ such that $\tau(t') = s$ with $(t', s) \in F$ and $(s, t) \in F$. In this case, we say that $Max(\beta)$ is consistent with the transition t.

A word $\sigma \in T^\omega$ is strict iff σ is strict with every transition $t \in alph(\sigma)$.

PROPOSITION 3.7. *Let Σ be a self-concurrency free system net and $\sigma \in \mathbf{F}(\Sigma)$. Then σ is strict iff there exists a process $\pi = (B, E, F', p)$ of Σ such that*

$$(E, \leq_\pi, p) = (O(\sigma), \leq_\sigma, lb).$$

PROOF. ($\Leftarrow$) Clearly, the injective mapping τ should be chosen corresponding to the relation F' of process π.
($\Rightarrow$) By the Construction 2.6 and the self-concurrency freeness of Σ (when we construct a process π associated to firing sequence σ by adding some new event $p^{-1}(t)$ corresponding to some transition t) we use the injective mapping τ of Definition 3.5 for connecting maximal conditions of current process with this event. So $(O(\sigma), \leq_\sigma, lb) \subseteq (E, \leq_\pi, p)$. The converse inclusion is obvious by Theorem 2.9. Hence $(E, \leq_\pi, p) = (O(\sigma), \leq_\sigma, lb)$. When σ is infinite, the proof follows from the relation $(\cup E_i, \cup \leq_{\pi_i}, \cup p_i) = (\cup O(\sigma_i), \cup \leq_{\sigma_i}, lb)$, where $\pi = \cup \pi_i, \pi_i = (E_i, B_i, F_i', p_i)$ and σ_i is the prefix of σ with $(E_i, \leq_{\pi_i}, p_i) = (O(\sigma_i), \leq_{\sigma_i}, lb)$.

Under the assumption of Proposition 3.7 the process π is called globally observable, and the corresponding semi-trace $[\sigma>$ is its global observation. We define the set of strict semi-traces and the set of strict traces on firing sequences corresponding by

$$SS(\Sigma) := \{[\sigma> \mid \sigma \in \mathbf{F}(\Sigma) \text{ and } \sigma \text{ is strict }\}.$$
$$ST(\Sigma) := \{[\sigma] \mid \sigma \in \mathbf{F}(\Sigma) \text{ and } \sigma \text{ is strict }\}.$$

By 2.11 and 3.2 the sets $SS(\Sigma)$ and $ST(\Sigma)$ have the same representatives. Now we give a sufficient condition for the converse of Theorem 3.4.

CORROLARY 3.8. *Let σ_1, σ_2 be strict firing sequences of self-concurrency free system net Σ and $\pi(\sigma_1) = \pi(\sigma_2)$. Then $\sigma_1 \equiv \sigma_2$.*

PROOF. There exist processes $\pi_1, \pi_2 \in \pi(\sigma_1) = \pi(\sigma_2)$ such that
$(E_1, \leq_{\pi_1}, p_1) = (O(\sigma_1), \leq_{\sigma_1}, lb)$ and $(E_2, \leq_{\pi_2}, p_2) = (O(\sigma_2), \leq_{\sigma_2}, lb)$.
So $Lin(\pi_1) = [\sigma_1 >, Lin(\pi_2) = [\sigma_2 >$. Since $\pi_1 \in \pi(\sigma_2)$ and $\pi_2 \in \pi(\sigma_1)$, $\sigma_2 \in [\sigma_1>$ and $\sigma_1 \in [\sigma_2>$. Hence $\sigma_1 \equiv \sigma_2$.

DEFINITION 3.9. A self-concurrency free system net Σ is globally observable iff all of its processes are globally observable.

THEOREM 3.10. *Let Σ be a globally observable system net. Then the mappings π_0 and Lin are order-preserving bijections between the set $SS(\Sigma)$ of strict semi-traces on firing sequences of Σ and the set of $\approx$-equivalence classes on processes of Σ, i.e.*

$$SS(\Sigma) \cong \mathbf{P}(\Sigma)/_{\approx},$$

where $\pi_0([\sigma>) = \{\pi \mid \pi \in \pi(\sigma) \wedge (O(\sigma), \leq_\sigma, lb) = (E, \leq_\pi, p)\}$.

PROOF. From the above presented results 2.8, 3.4, 3.7 and 3.8 we need only to show that if $Lin(\pi) = [\sigma>$, then $(O(\sigma), \leq_\sigma, lb) = (E, \leq_\pi, p)$. In this case, the word σ is strict by Proposition 3.7. From the self-concurrency freeness of Σ for $\pi = (B, E, F', p)$ with $Lin(\pi) = [\sigma>$ there is a unique bijection *name*: $E \to O(\sigma)$ such that $p(e) = lb(name(e))$. So we can identify the set of events of the process π and the set $O(\sigma)$. Hence, by 2.9 and since $Lin(\pi) = [\sigma>$ we get $(O(\sigma), \leq_\sigma, lb) = (E, \leq_\pi, p)$.

By Theorem 3.10, strict semi-traces on firing sequences can replace processes of globally observable nets. Since for 1-safe nets semi-commutation and commutation coincide and since 1-safe nets are globally observable. Theorem 3.10 extends the results on the relationship between traces and processes of 1-safe nets in [2,3,8].

EXAMPLE 3.11. Let Σ be the system net given in Fig. 3. It is easy to see that Σ is globally observable.

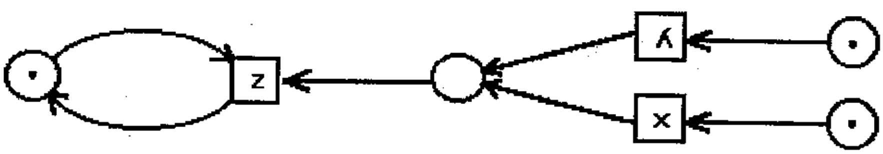

Fig. 3

The set of strict semi-traces is

$$\{[x>, [y>, [xy>, [xz>, [yz>, [xzy>, [yzx>, [xzyz>, [yzxz>\}.$$

As labelled partial ordered sets, they are isomorphic to the corresponding subprocesses of the two processes shown in Fig. 4.

4. Regular property of the set of processes

In this section we consider the relationship between firing sequences and processes by distinguishing the individualities of several tokens in the same place. This relationship can be expressed by a 1-safe system net constructed from a given n-safe system net, which has, in a certain sense, the same set of processes as the original net. Combining with the results in [2, 3] we get a

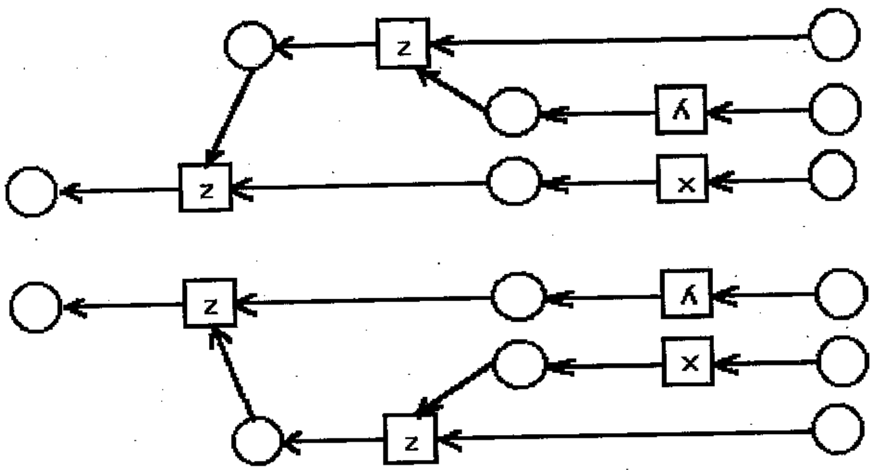

Fig. 4

regular characterization of the set of processes of the original net. When we deal with some classes of nets as self-concurrency free n-safe or globally observable nets, then the regularity of the set of firing sequences implies the regularity of the set of processes as labelled partial orderings derived from the representatives of regular trace language.

CONSTRUCTION 4.1. Let $\Sigma = (S, T, F, M_0)$ be an n-safe system net with $k(s) = max\{M(s) \mid \forall M \in [M_0\rangle\} \leq n$ for every place $s \in S$. Define a 1-safe system net $\Sigma' = (S', T', F', M_0')$ as follows:

(i) $S' = \cup_{s \in S}\{s_1, s_1', \ldots, s_{k(s)}, s_{k(s)}'\}$,

(ii) $T' = \cup\{(t, \{s_{i(s)} \mid s \in {}^\bullet t \cup t^\bullet\}) \mid t \in T \;\wedge\; i(s) \leq K(s) \;\wedge\; i(s) \in \mathbf{N}\}$,

(iii) For $s_i \in S', (t, B) \in T'$.

$(s_i, (t, B)) \in F'$ iff $s_i \in B \;\wedge\; s \in {}^\bullet t$,

$((t, B), s_i) \in F'$ iff $s_i \in B \;\wedge\; s \in t^\bullet$,

$(s_i', (t, B)) \in F'$ iff $s_i \in B \;\wedge\; s \in t^\bullet \wedge s \notin {}^\bullet t$,

$((t, B), s_i') \in F'$ iff $s_i \in B \;\wedge\; s \in {}^\bullet t \wedge s \notin t^\bullet$,

(iv) $M_0'(s_i) = 1$ iff $M_0(s) > 0 \;\wedge\; i \leq M_0(s)$,

$M_0'(s_i') = 1$ iff $M_0(s_i) = 0$,

$M_0'(p) = 0$ for the other cases of p.

By this construction each place s with maximal number of tokens k in Σ is multiplied into k places in Σ' indexed by $1, 2, \ldots, k$. Each transition t in Σ is multiplied into transitions in Σ' in order to distinguish token flows. There is an arc from (to) s_i to (from) a transition in Σ' corresponding to the transition t in Σ iff there is an arc from (to) s to (from) t in Σ. The places s' are introduced to ensure the 1-safeness of Σ'.

By definition, Σ' is a 1-safe, countable and degree-finite net.

EXAMPLE 4.2. Let us consider a 2-safe system net Σ given in Fig. 1. The associated 1-safe system net Σ' is shown in Fig. 5.

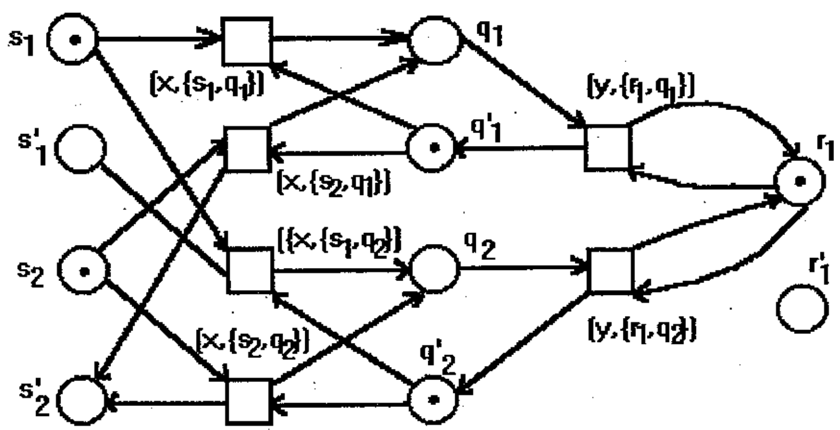

Fig. 5

Let l: $\Sigma' \to \Sigma$ be defined as $l((t,B)) = t$, $l(s_i) = s$, $l(s_i') =$ "undefined".

For each process $\pi = (B_1, E_1, F_1, p_1)$ of Σ', we define $h(\pi) = (B_2, E_2, F_2, p_2)$, where $B_2 = B_1 \setminus \{b \mid p_1(b) = s_i'\}$, $E_2 = E_1$, $F_2 = F_1|_{B_2 \times E_1 \cup E_1 \times B_2}$, $p_2 = l \circ p_1|_{B_2 \cup E_1}$.

PROPOSITION 4.3. *Let Σ be an n-safe system net and Σ' the associated 1-safe system net with the mapping h:* $\mathbf{P}(\Sigma') \to \mathbf{P}(\Sigma)$. *Then*

(i) h is a surjective mapping.

(ii) h is bijection iff Σ is a self-concurrency free net.

PROOF. (i) has been proved by using Constructions 2.6 and 4.2 in [7].

(ii) By (i) and Theorem 2.9 there exists, for every process $\pi \in P(\Sigma)$, a firing sequence σ' of Σ' such that $Lin(\pi) = l(\{\sigma" \mid \sigma" \equiv \sigma'\})$. So there are two firing sequences $\gamma', \gamma"$ of Σ' such that $\gamma' \not\equiv \gamma"$ and $l(\gamma') = l(\gamma")$ if and only if there exist $x, y, y' \in T'^*$ and $(t, B), (t, B') \in T'$ for which $\gamma' = x(t, B)y$, $\gamma" = x(t, B')y'$ and $B \neq B'$. Thus, $\{t, t\}$ are enabled at a reachable marking M: $M_0[l(x)> M$, i.e., Σ is not a self-concurrency free net, a contradiction.

Let Σ be an n-safe system net. The associated system net Σ' is a 1-safe system net. There is an one-to-one corresponding between processes and $\equiv$-equivalence classes on firing sequences of Σ'. Here we notice that the relation $\equiv$ defined in 3.3 restricted to 1-safe system nets is exactly the relation introduced by Mazukiewicz [8]. The set of $\equiv$-equivalence classes of Σ', as well-known, forms a regular trace language which can be recognized by a finite asynchronous automaton (see [13]). By 4.3 we get the set of processes of Σ from the trace language on firing sequences of Σ'. This can be formulated formally by the following theorem.

THEOREM 4.4. *Let $\Sigma = (S, T, F, M_0)$ be a self-concurrency free n-safe system net. Then there exist a commutative system (A, R), a mapping l: $A \to T$ and a prefix closed regular trace language L over $<A, R>$ with order-preserving bijection ϕ: $L \longleftrightarrow \mathbf{P}(\Sigma)/_{\approx}$ between the set of labelled partial orderings derived from traces of L and the set of labelled partial orderings derived from processes of Σ such that*

$$\phi(\sigma') = \pi = (E, B, F', p) \Leftrightarrow (O(\sigma'), \leq_{\sigma'}, l \circ lb) = (E, \leq_\pi, p).$$

In [6] the regularity of the set of maximal traces is studied in connection with the regularity of the set of firing sequences. Here we investigate regular properties of the set of strict traces on firing sequences.

THEOREM 4.5. *Let Σ be a self-concurrency free system net. If $\mathbf{F}(\Sigma)$ is a regular language, then $ST(\Sigma)$ is a regular trace language.*

PROOF. Suppose a finite asynchronous automaton $\mathcal{A} = (Q, T, \delta, q^0, F)$ recognizes the trace language $[\mathbf{F}(\Sigma)]$, where

- $Q = (Q_1, \dots, Q_n)$ with Q_i is a finite set for $1 \leq i \leq n$.
- If J is a subset of $\{1, \dots, n\}$, we denote by Q_J the product $\prod_{j \in J} Q_j$, and if q is an element of Q, q_J will be the element of Q_J consisting of the components of q having their index in J.
- I: $T \to 2^{\{1,2,\dots,n\}}$.
- For each letter t there is a corresponding mapping δ_t: $Q_{I(t)} \to Q_{I(t)}$.
- The transition function δ: $Q \times T \to Q$ is defined as follows: $q' = \delta(q, t)$ is the unique state q' such that $q'_{I(t)} = \delta_t(q_{I(t)})$, and $\forall j \notin I(t)$, $q'_j = q_j$.

- F is a subset of Q, the set of final states, and q^0 is an element of Q, the initial state.

Because the class of all recognizable trace languages is closed under intersection, it is sufficient to prove that the set $ST_t(\Sigma) = \{[\sigma] \mid \sigma \in F(\Sigma)$ and σ is strict with the transition $t \in T\}$ is a regular trace language. Then so is $ST(\Sigma) = \bigcap ST_t(\Sigma)$.

Now we construct a finite asynchronous automaton $\mathcal{A}'_t = (Q', T, \delta', q^{0'}, F')$ that recognizes $ST_t(\Sigma)$ for the transition $t \in T$ as follows:

$Q' = (Q'_1, \dots, Q'_n)$ and $Q'_i = Q_i$ for $i \notin I_t$, $Q'_i = (Q_i, 2^{T \cup \{\Lambda\}})$ for $i \in I_t$.

$q^{0'} = (q_1^{0'}, \dots, q_n^{0'})$ with $q_i^{0'} = q_i^0$ for $i \notin I_t$ and $q_i^{0'} = (q_i^0, \emptyset)$ for $i \in I_t$.

For $q'_i = q_i$ we denote $first(q'_i) = q_i$ and $last(q'_i)$ ="undefined" and for $q'_i = (q_i, H)$ with $H \subseteq T \cup \{\Lambda\}$, $first(q'_i) = q_i$ and $last(q'_i) = H$.

We define the set of final states F' and the state function δ' by

$F' = \{(q'_1, \dots, q'_n) \mid (first(q'_1), \dots, first(q'_n)) \in F$ and $\forall i \in I_t \;\; last(q'_i) \neq \{\Lambda\}\}$.

For a transition $a \in T$ we set $\delta'(q'_1, \dots, q'_n, a) = (q_1", \dots, q_n")$ where $(first(q_1"), \dots, first(q_n")) = \delta(first(q'_1), \dots, first(q'_n), a)$. The second component $last(q_i")$ for $i \in I_t$ is defined by

(i) $last(q_i") = last(q'_i)$ if $a \neq t$ and $a^\bullet \cap {}^\bullet t = \emptyset$.

(ii) $last(q_i") = last(q'_i) \setminus \{b \mid b^\bullet \cap {}^\bullet a \neq \emptyset\} \cup \{a\}$ if $a \neq t$, $a^\bullet \cap {}^\bullet t \neq \emptyset$ and $last(q'_i) \neq \{\Lambda\}$.

(iii) $last(q_i") = \emptyset$ if $a = t$, $last(q'_i) \neq \{\Lambda\}$ and $last(q'_i)$ is consistent with t.

(iv) $last(q'_i) = \{\Lambda\}$ for the other cases, i.e., if $last(q'_i) = \{\Lambda\}$ or if $a = t$ and $last(q'_i)$ is not consistent with t.

The element Λ is introduced to indicate that a state containing a component with this element corresponds to some firing sequence which is not consistent with t. It is easy to verify that the constructed finite asynchronous automaton $\mathcal{A}'_t$ recognizes the trace language $ST_t(\Sigma)$.

By Theorems 3.10 and 4.5, the regularity of the set of firing sequences for a globally observable net implies the regularity of the set of its processes.

For general nets, E.Ochmanski [9], Hung and Knuth [6] suggested the set of maximal semi-traces for representing their concurrent behaviour. From the results presented in this paper we would like to propose the set of strict semi-traces on firing sequences as some domain [12] or labelled partial orderings [7] for representing concurrent behaviour of some classes of nets. This paper is the extended version of [7].

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