ACTA MATHEMATICA VIETNAMICA
Vol. 13 No 2 (1988)

ON VON NEUMANN REGULAR RINGS, XV

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INTRODUCTION

Von Neumann regular rings and associated rings (for example, self—injective regular rings, V-rings and generalizations) have drawn considerable attention in recent years (cf. [1], [2], [6], [7], [10], [11], [12] and the bibliography of [5]). In this sequel to [17] and [18], the following results are considered: (1) If A is a ring whose essential left ideals are two-sided ideals, the following are equivalent: (a) A is von Neumann regular; (b) A is semi-prime such that every essential left ideal is idempotent: (c) A is semi-prime such that every simple left A-module is flat; (2) A is strongly regular if, and only if, every simple right A-module is flat and every complement left ideal of A is a two-sided ideal; (3) A semi-prime left CM-ring whose principal left ideals are complement left ideals is either strongly regular or semi-simple Artinian; (4) A left duo left non-singular ring A is left Goldie iff every p-injective torsionfree left A-module is injective; (5) If A is semi-prime left Goldie, then every divisible torsionfree left A-module is C-projective; (6) A commutative ring whose divisible torsionfree quasi-injective A-modules are p-injective admits a von Neumann regular classical quotient ring. Self-injective regular rings are also studied.

Throughout the paper, A represents an associative ring with identity and A-modules are unital. For general terminology (in particular, concerning classical quotient rings), consult [4]. In this note, J, Z, S denote respectively the Jacobson radical, left singular ideal and left socle of A. A left (right) ideal of A is called reduced if it contains no non-zero nilpotent element. An ideal of A will always mean a two-sided ideal and following E. H. FELLER A is called left duo if every left ideal is an ideal. As an analogy to the characterization of semi-simple Artinian rings as rings whose left (right) modules are injective, von Neumann regular rings may be characterized as rings whose left (right) modules are p-injective. (A left A-module M is called p-injective if, for any principal left ideal p of A, every left A-homomorphism of p into M extends to A.) Von Neumann regular rings are also called « absolutely flat » (in the sense that all modules are flat [8, p. 262]). For an arbitrary ring A, there is no inclusion relation between the classes of flat A-modules and p-injective A-modules. A is called left p-injective iff ${}_AA$ is p-injective. A result of M. IKEDA — T. NAKAYAMA asserts that A is left p-injective iff every principal right ideal of A is a right annihilator. Left p-injective rings effectively generalize left self-injective rings, von Neumann regular rings and right pseudo-Frobeniusean rings. K. VARADARAJAN and K. WEHRHAHN have studied p-injectivity in connection with torsion theories [11].

As before, A is ELT (resp. MELT) if every essential (resp. maximal essential, if it exists) left ideal of A is an ideal of A. If A is semi-prime, then S is also the right socle of A. A ring is called semi-simple if its Jacobson radical is zero. Thus A is semi-simple iff $J = 0$. A is called fully idempotent (resp. fully left idempotent, fully right idempotent) if every ideal (resp. left ideal, right ideal) of A is idempotent.

PROPOSITION 1. *The following conditions are equivalent*:

(1) *A is ELT regular;*

(2) *Every essential left ideal of A is an essential right ideal and every essential right ideal of A is idempotent;*

(3) *A is semi-prime such that every essential left ideal is an idempotent ideal of A;*

(4) *A is a semi-prime MELT ring whose simple left modules are flat;*

(5) *Every factor ring of A is a semi-prime MELT left p-injective ring.*

Proof. If A is semi-prime, then any essential left ideal of A which is an ideal of A must be right essential. Therefore (1) implies (2).

Assume (2). Let T be an ideal of A such that $T^2 = 0$. Then $r(T)$ is an essential left ideal of A which is therefore right essential by hypothesis. The set of right subideals of $r(T)$ having zero intersection with T has, by Zorn's Lemma a maximal member C which is a complement right subideal of $r(T)$ such that $E = T \oplus C$ is an essential right subideal of $r(T)$. Then E is an essential right ideal of A which is therefore idempotent by hypothesis. Now $E = E^2 = (T \oplus C)(T \oplus C) \subseteq (T \otimes C)C$ which yields $T \subseteq E = (T \oplus C)C$, whence $T = TC = 0$. This proves that A is semi-prime and thus (2) implies (3).

Assume (3). For any ideal T of A, there exists a complement left ideal K such that $L = T \oplus K$ is an essential left ideal of A. Since $L = L^2$, then $T \subseteq (T \oplus K)T \subseteq T^2 + KT$. But $TK = 0$ implies $(KT)^2 = 0$, whence $KT = 0$. Thus $T = T^2$ which proves that A is fully idempotent. If B is a prime factor ring of A, then every essential left ideal of B is an idempotent ideal. For any $0 \neq u \in B$, BuB is an essential left ideal of B and if C is a complement left subideal of BuB such that $E = Bu \oplus C$ is an essential left subideal of BuB, then E is essential in ${}_BB$ which implies $E = E^2$. Since $u \in E^2$, then $u \in BuBu$ which proves that B is a fully left idempotent ring. Therefore B is regular by [1, Theorem 3.1] It follows from [5, Corollary 1.18] that A is regular and hence (3) implies (4).

Assume (4). Set $B = A/S$, where S is the socle of A. Then every simple left B-module is flat and every maximal left ideal of B is an ideal of B. Let $\overline{B} = B/J(B)$, where $J(B)$ is the Jacobson radical of B. Then $\overline{B}$ is a semi-simple ring whose simple left modules are flat and whose maximal left ideals are ideals. Since any semi-simple ring whose maximal left ideals are ideals must be reduced, then $\overline{B}$ is a reduced ring whose simple left modules are flat which implies that $\overline{B}$ is a strongly regular ring. Therefore every maximal right ideal of B is an ideal of B. By [14, Theorem 1.7], B is strongly regular which implies that A is *ELT*. Now since A is semi-prime, then S is a fully left and right idempotent ring which, together with A/S fully left and right idempotent, implies that A is also a fully left and right idempotent ring. By [7, Corollary 6], A is regular and hence (4) implies (5).

Assume (5). Then A is a fully idempotent ring and A/S is a semi-simple ring whose maximal left ideals are ideals, whence A/S is a reduced ring. Since A/S is left p-injective, then A/S is strongly regular. Therefore A is an ELT fully left and right idempotent ring which is now regular and finally (5) implies (1).

Applying [16, Theorem 1], we get

COROLLARY 1.1. *The following conditions are equivalent:*

(1) *Every factor ring of A is an ELT left and right self-injective regular, left and right V-ring of bounded index ;*

(2) *Every factor ring of A is a semi-prime MELT left self-injective ring.*

Remark 1. Suppose that every essential left ideal of A is idempotent and every complement one-sided ideal of A is an ideal of A. Then A is a reduced fully left and right idempotent ring such that the maximal left and right quotient rings of A coincide. (cf. [4, Theorem 2. 38] and [18, Proposition 1].)

A right A-module M is called f-injective if, for any finitely generated right ideal F of A, every right A-homomorphism of F into M extends to A. Of course, f-injectivity coincides with P-injectivity over von Neumann regular rings.

PROPOSITION 2. *The following conditions are equivalent for an ELT ring A :*

(1) *A is a left and right self-injective regular, left and right V-ring of bounded index ;*

(2) *Every maximal right ideal of A is f-injective and every finitely generated non-singular left A-module is projective.*

Proof. It is known that (1) implies (2) (cf. [19, Corollary 6])

Assume (2). Suppose $Z \neq 0$. Then Z contains a non-zero z such that $z^2 = 0$. Since $l(z)$ is an ideal of A. let M be a maximal right ideal of A containing $l(z)$. Since M_A is f-injective, the inclusion map $zA \to M$ yields $z = wz$ for some $w \in M$ whence $1 - w \in l(z) \subseteq M$, yielding $l \in M$, which contradicts $M \neq A$. This proves that $Z = 0$. If F is a finitely generated proper right ideal of A, R a maximal right ideal containing F, the inclusion map $F \to R$ yields $(1 - u)F = 0$ for some $u \in R$, showing that the left annihilator of F is non-zero. A result of H. BASS then says that a finitely generated projective submodule of any projective let A-module is always a direct summand. Suppose that Q, the maximal left quotient ring of A, is different from A. If $q \in Q$, $q \notin A$, then $B = A + Aq$ is a finitely generated non-singular left A-module which is therefore projective. Then ${}_AA$ is a direct summand of ${}_AB$. But ${}_AA$ is essential in ${}_AB$ which yields $A = B$, contradicting $q \notin A$. Thus $A = Q$ is left self-injective and (2) implies (1) by [16, Theorem 1].

It is well-known that strongly regular rings are left and right duo. We give some new characteristic properties of strongly regular rings.

PROPOSITION 3. *If every complement left ideal of A is an ideal, the following conditions are then equivalent :*

(1) *A is strongly regular;*

(2) *Every maximal right ideal of A is p-injective;*

(3) *Every simple right A-module is flat;*

(4) *A is a left p-injective ring whose simple left modules are flat.*

Proof. Obviously, (1) implies (2).

If I is a p-injective right ideal of A, then A/I is a flat right A-module. It follows that (2) implies (3).

Assume (3). By [18, Proposition 1], $B = A/Z$ is a reduced ring. Since every simple right A-module is flat, then every simple right B-module is flat which yields B strongly regular. Now for any $z \in Z$, $a \in A$, setting $u = 1 - za$, we have $l(u) = 0$. If $uA \neq A$, let M be a maximal right ideal containing aA. Since A/M_A is flat, then $u = wu$ for some $w \in M$ and therefore $1 - w \in l(u) = 0$ which implies $1 \in M$, contradicting $M \neq A$. This proves that u is right invertible in A and hence $z \in J$, yielding $Z \subseteq J$. Let K be any maximal left ideal of A. Since $Z \subseteq J \subseteq K$, and K/Z is an ideal of B, then K is an ideal of A. Then A is strongly regular by [14, Theorem 1.7] and therefore (3) implies (4).

Assume (4). Since A is left p-injective, then $J = Z$ [17, Proposition 4]. The above argument then shows that every maximal right ideal of A is an ideal and therefore (4) implies (1).

Note that if every simple left A-module is flat, then for any $z \in Z$, $Z + A(1 - z) = A$. Consequently, if ${}_AZ$ is superfluous in ${}_AA$, then $Z \subseteq J$.

Question. Is A strongly regular if every complement left ideal of A is an ideal and every simple left A-module is flat?

As usual, A is called left uniform if every non-zero left ideal is an essential left ideal. If A is left uniform, then every complement left ideal of A is an ideal. Recall that A is a left CM-ring [15] if, for any maximal essential left ideal M of A (if it exists), every complement left subideal of M is an ideal of M. Left CM-rings generalize left uniform rings, left duo rings, left PCI rings [2] and semi-simple Artinian rings. Since a prime left CM-ring is either simple Artinian or left uniform, the next corollary then follows immediately.

COROLLARY 3.1. *A is simple Artinian if A is a prime left CM-ring whose simple right modules are flat.*

The next property of self-injective rings seems new.

Remark 2. If A is a left self-injective ring such that every complement left ideal is an ideal, then the maximal left ideals of A coincide with the maximal right ideals of A. (In that case, the following conditions are equivalent for any maximal left ideal M of A: (a) ${}_A A/M$ is injective; (b) ${}_A A/M$ is p-injective; (c) A/M_A is flat.

We now have a further result on CM-rings.

PROPOSITION 4. *The following conditions are equivalent:*

(1) *A is either strongly regular or semi-simple Artinian;*

(2) *A is a semi-prime left CM-ring whose principal left ideals are complement left ideals;*

(3) *A is a left non-singular, left CM-ring such that each prime factor ring of A is left p-injective.*

Proof. It is easily seen that (1) implies (2) and (3).

Assume (2). Suppose that $Z \neq 0$. Then there exists $0 \neq z \in Z$ such that $z^2 = 0$. Let K be a complement left ideal of A such that $L = Az \oplus K$ is an essential left ideal of A. Since Z cannot contain any non-zero idempotent, then $L \neq A$. If M is a maximal left ideal of A containing L, then M is a maximal essential left ideal of A. Since A is left CM, then $AzM \subseteq Az$ which yields $(Mz)^2 \subseteq AzMz \subseteq Az^2 = 0$, whence $M = l(z)$ (A being semi-prime). Therefore Az ($\approx A/M$) is a minimal left ideal of A which is therefore generated by a non-zero idempotent, contradicting $Az \subseteq Z$. This proves that $Z = 0$. Suppose that A is not semi-simple Artinian. Then A is reduced by [15, Lemma 1.6 (1)]. Also, every principal left ideal of A is a left annihilator [15, Remark 2(2)]. Therefore A is strongly regular and (2) implies (1).

Assume (3). Since A is left non-singular left CM, then A is either semi-simple Artinian or reduced [15, Lemma 1.6 (1)]. In the latter case, every completely prime factor ring of A, being p-injective, is a division ring and hence A is strongly regular. Thus (3) implies (1).

At this point, we turn to rings having classical quotient rings. For definitions and results on classical quotient rings, consult, for example, [4]. Projective and injective modules are fundamental concepts in ring theory (cf. [3], [4], [8]). Recall that a left A-module M is divisible iff $M = cM$ for each non-zero-divisor c of A. We know that p-injective left A-modules are divisible [13, p. 176] but the converse is not true, ${}_A M$ is called torsionfree if, for any $0 \neq Y \in M$ and any non-zero-divisor c of A, $cy \neq 0$.

The next two propositions are motivated by an important result of L. LEVY [9, Theorem 3.3] which states that if A has a classical left quotient ring Q, then Q is semi-simple Artinian iff every divisible torsionfree left A-module is injective.

PROPOSITION 5. *The following conditions are equivalent:*

(1) A is semi-simple Artinian:

(2) A has a classical left quotient ring and every divisible torsionfree quasi injective left A-module is projective.

Proof. Obviously, (1) implies (2).

Assume (2). Let A have a classical left quotient ring Q and let P be a quasi-injective left Q-module. Then ${}_AP$ is divisible torsionfree and if ${}_AH$ is a submodule of ${}_AP$, $f: H \to P$ a left A-homomorphism, $i: H \to P$ the inclusion map, we may define a left Q-homomorphism $F: {}_QQH \to {}_QP$ by $F(\sum_{i=1}^{m} c_i^{-1} d_i) = \sum_{i=1}^{m} c_i^{-1} f(d_i)$, for all $d_i \in H$, c_i non-zerodivisors in A. Since ${}_QP$ is quasi-injective, there exists $T: {}_QP \to {}_QP$ such that $Ti = F$. This shows that f extends to an endomorphism of ${}_AP$ yielding ${}_AP$ quasi-injective. By hypothesis, ${}_AP$ is projective. Now if $g: {}_QP \to {}_QN$ is a left Q-homomorphism, $P: {}_QM \to {}_QN$ an epimorphism, let $\widehat{g}$, $\widehat{p}$ be the restrictions of g, p respectively to ${}_AP$, ${}_AM$. Then there exists a left A-homomorphism $\widehat{h}: {}_AP \to {}_AM$ such that $\widehat{p}\widehat{h} = \widehat{g}$. If $h: {}_QP \to {}_QM$ is the left Q-homomorphism defined by $h(qy) = q\widehat{h}(y)$ for all $q \in Q$, $y \in P$, then for any $u \in p$, $ph(u) = p(h(u)) = p\,(\widehat{h}(u)) = \widehat{p}(\widehat{h}(u)) = \widehat{g}(u) = g(u)$ which proves that $ph = g$. Thus ${}_QP$ is projective. Therefore every simple left Q-module (being quasi-injective) is projective which implies that Q is semi-simple Artinian. A is then a semi-prime left Goldie ring such that ${}_AQ$ is quasi-injective and therefore projective. This yields A semi-simple Artinian [4, P. 102] and hence (2) implies (1).

COROLLARY 5, 1 *The following conditions are equivalent:*

(1) A is a finite direct sum of division rings;

(2) A is a left duo ring whose divisible torsionfree quasi-injective left modules are projective.

Even for commutative rings, divisible modules need not be p-injective.

PROPOSITION 6. *The following conditions are equivalent for a left duo left non-singular ring A:*

(1) A is left Goldie;

(2) Every p-injective torsionfree left A-module is injective.

Proof. Since A is left duo, then A possesses a classical left quotient ring Q. Since A is left duo left non-singular, then A is reduced which implies that Q is a reduced ring. If A is left Goldie, then Q is semi-simple Artinian and since every p-injective left A-module is divisible [13, p. 176], then (1) implies (2) by [9, Theorem 3.3]

Assume (2). Let M be a p-injective left Q-module. If P is a principal left ideal of A, $f : P \to M$ a left A-homomorphism, define $g : QP \to M$ by $g\left(\sum_{i=1}^{m} q_i p_i\right) = \sum_{i=1}^{m} q_i f_i(p_i)$ for all $q_i \in Q$, $p_i \in P$. Then g is a well-defined left Q-homomorphism and since QP is a principal left ideal of Q, there exists $y \in M$ such that $g(qp) = gpy$, for all $q \in Q$, $p \in P$. Therefore, for any $p \in P$, $f(p) = g(p) = py$ which proves that ${}_AM$ is p-injective. Since ${}_AM$ is torsionfree, then ${}_AM$ is injective. If L is an essential left ideal of Q, $h : L \to M$ a left Q-homomorphism, setting $I = L \cap A$. then $QI = L$ and if $r : {}_AI \to {}_AM$ is the left A-homomorphism defined by $r(ab) = ah(b)$ for all $a \in A$, $b \in I$, there exists $z \in M$ such that $r(b) = bz$ for all $b \in I$. Then for any $q \in Q$, $b \in I$, $h(qb) = qh(b) = qr(b) = qbz$ which implies that $h(t) = tz$ for all $t \in L$. Thus ${}_QM$ is injective. Since any direct sum of p-injective left Q-modules is p-injective, then Q is left Noetherian by [3, Theorem 20.1]. Therefore Q is a reduced left Noetherian ring which is its own classical left quotient ring. It follows that Q is semi-simple Artinian and (2) implies (1).

COROLLARY 6. 1. *If A is commutative semi-prime, then A is Goldie iff every p-injective torsionfree A-module is injective.*

Although injective modules are p-injective, it is clear that quasi-injective modules need not be p-injective.

The proofs of Propositions 5 and 6 yield the next result.

PROPOSITION 7. *If A is commutative such that every divisible torsionfree quasi-injective A-module is p-injective, then A possesses a von Neumann regular classical quotient ring.*

Following [17], a left A-module P is called C-projective if, for any cyclic left A-modules M, N with an epimorphism, $g : M \to N$, an yleft A-homomorphism $f : P \to N$, there exists a left A-homomorphism $h : P \to M$ such that $gh = f$.

PROPOSITION 8. *Let A have a classical left quotient ring Q. If P is a C-projective left Q-module, then ${}_AP$ is C-projective.*

Proof. Let M, N be cyclic left A-modules, $f : P \to N$ a left A-homomorphism, $g : {}_AM \to {}_AN$ an epimorphism. Define $G : QM \to QN$ by $G(qw) = qg(w)$ for all $q \in Q$, $w \in M$. If $N = Av$, there exists $u \in M$ such that $g(u) = v$. Since $QN = Qv$, then $G(u) = g(u) = v$ and for any $q \in Q$, $G(qu) = qg(u) = qv$ which shows that G is an epimorphism of left Q-modules. If $F : {}_QP \to {}_QQN$ is the left Q-homomorphism defined by $F(qp) = qf(p)$ for all $q \in Q$, $p \in P$, since ${}_QP$ is C-projective, there exists a left Q-homomorphism $H : P \to QM$ such that $GH = F$. Since, for each $p \in P$, $F(p) = f(p) \in N$, then $H(p) \subseteq M$ and there exists a left A-homomorphism $h : P \to M$ defined by $h(p) = H(p)$ for all $p \in P$ such that $gh = f$. This proves that ${}_AP$ is C-projective.

COROLLARY 8.1. *If A is semi-prime left Goldie, then every divisible torsionfree left A-module is C-projective. (If Q is the classical left quotient ring of A, then every divisible torsionfree left A-module is a left Q-module [9, p. 140].)*

Applying Proposition 5, we get

COROLLARY 8.2. *A is semi-simple Artinian iff A is a semi-prime left Goldie ring whose quasi-injective C-projective left modules are projective.*

This corollary shows that quasi-injective C-projective left A-modules need not be projective. In particular, C-projectivity is weaker than projectivity. However, if A is von Neumann regular, then projectivity coincides with C-projectivity [17, Corollary 2.3].

[6, Theorem 8] and [17, Proposition 1] yield a connection between C-projectivity and p-injectivity.

Remark 3. If every idempotent of A is central, the following conditions are equivalent: (a) every divisible left A-module is p-injective; (b) every principal left ideal of A is C-projective; (c) every principal left ideal of A is projective. In that case, A is reduced and three more equivalent conditions are obtained by replacing « left » by « right » in (a), (b), (c).

If A is a principal left ideal ring, then injective left A-modules coincide with p-injective left A-modules. But the converse is not true (otherwise, any semi-prime left hereditary, left and right Goldie ring would be a principal left ideal ring!).

Remark 4. If A is a semi-prime left Goldie ring whose left ideals are C-projective, then A is left hereditary left Noetherian. (Apply [9, Theorem 3.11] and [17, Corollary 2.1].)

Pseudo-Frobeniusean rings are extensively studied in [8]. The next remark is motivated by [17, Question 1].

Remark 5. If A is left pseudo-Frobeniusean, then any C-projective left A-module is projective.

Remark 6. Let A be a left Noetherian ring. Then A is left Artinian iff each prime factor ring of A is left p-injective.

Remark 7. The following conditions are equivalent: (1) A is right self-injective regular; (2) A is a right f-injective right non-singular ring such that the injective hull of A_A is a projective right p-module.

Let me conclude with a last remark on regular rings and semi-simple Artinian rings.

Remark 8. (1) A is von Neumann regular iff every principal left ideal of A is the flat left annihilator of an element of A; (2) The following conditions are equivalent: (a) A is semi-simple Artinian; (b) every left ideal of A is the flat left annihilator of an element of A; (c) A is a right principal ideal ring such that every principal left ideal is a flat annihilator.

REFERENCES

[1] G. BACCELLA, *Generalized V-rings and von Neumann regular rings*, Rend. Sem. Mat. Univ. Padova 72 (1984), 117-133.

[2] A. K. BOYLE, *Hereditary QI-rings*, Trans. Amer. Math. Soc. 192 (1974), 115 – 120.

[3] C. FAITH, *Algebra II : Ring Theory*, Springer-Verlag (1976)

[4] K. R. GOODEARL, *Ring Theory : Non singular rings and modules*, Pure Appl. Maths, n^0 33, Dekker (New York) (1976).

[5] K. R. GOODEARL, *Von Neumann regular rings*, Pitman (London) (1979).

[6] Y. HIRANO, M. HONGAN and M. ÔHORI, *On right p. p. rings*. Math. J. Okayama Univ, 24 (1982), 99 – 109.

[7] Y. HIRANO and H. TOMINAGA, *Regular rings V-rings and their generalizations*, Hiroshima Math. J. 9 (1979), 137 – 149.

[8] F. KASCH, *Modules and rings*, London Math. Soc. Monograph 17 (translated by D.A.R. WALLACE) (1982).

[9] L. LEVY, *Torsionfree and divisible modules over non-integral domains*, Canad. J. Math. 15 (1963), 132 – 151.

[10] K. VARADARAJAN, *Generalized V-rings and torsion theories*, Comm. Algebra 14 (1986), 455 – 467.

[11] K. VARADARAJAN and K. WEHRHAHN, *P-injectivity of simple pre-torsion modules*, Glasgow Math. J. 28 (1986), 223 – 225.

[12] Y. TAKEHANA, *V-rings relative to hereditary torsion theories*, Tsukuba J. Math. 6 (1982), 293 – 298.

[13] R. YUE CHI MING, *On simple p-injective modules*, Math. Japonica 19 (1974), 173 – 176.

[14] R. YUE CHI MING, *On von Neumann regular rings*, V. Math. J. Okayama Univ, 22 (1980), 151 — 160.

[15] R. YUE CHI MING, *On regular rings and self-injective rings*. Monatshefte für Math. 91 (1981), 153 – 166.

[16] R. YUE CHI MING, *On self-injectivity and strong regularity*. Acta Sci. Math. 47 (1984), 227 – 288.

[17] R. YUE CHI MING, *On von Neumann regular rings*, XIII. Ann. Univ. Ferrara Sez. VII Sci. Mat. 31 (1985), 49 – 61.

[18] R. YUE CHI MING, *On von Neumann regular rings*, XIV. Anal. Univ. Timisoara Ser. Mat. (to appear).

[19] J. ZELMANOWITZ, *Injective hulls of torsionfree modules*. Canad. J. Math. 23 (1971), 1094 — 1101.

Received November 30, 1987

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