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INTEGRAL REPRESENTATIONS OF THE SOLUTION OF SOMÉ HYPERBOLIC SYSTEMS WITH DEGENERATE COEFFICIENTS AND THEIR APPLICATIONS

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The aim of the present paper is to establish the integral representations and the inversion formulas for a new class of functions which will be called (p, q)-wave functions. The results are used to solve some boundary value problems for these functions.

1. THE (P, Q)-WAVE FONCTIONS

Lut us denote by G a region in the plane of variable x, y.

DEFINITION. A function

$$F(z) = U(x,y) + iV(x,y) \tag{1}$$

is said to be *(p,q)-wave* in the region G if $U(x, y), V(x,y) \in C^1(G)$ and the following conditions are satisfied:

$$p\frac{\partial U}{\partial x} - q\frac{\partial U}{\partial y} - \frac{\partial V}{\partial y} = 0,$$
$$q\frac{\partial U}{\partial x} - p\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} = 0. \tag{2}$$

where $p = p(x,y)$ and $q = q(x,y)$ are given real functions of the class $C(G)$, $z = x + iy$.

Note that it is always possible to find some transformations of the independent variables such that any linear hyperbolic system

$$a_1\frac{\partial U}{\partial \xi} + a_2\frac{\partial U}{\partial \eta} + a_3\frac{\partial V}{\partial \xi} + a_4\frac{\partial V}{\partial \eta} = 0,$$
$$b_1\frac{\partial U}{\partial \xi} + b_2\frac{\partial U}{\partial \eta} + b_3\frac{\partial V}{\partial \xi} + b_4\frac{\partial V}{\partial \eta} = 0.$$

where $a_j(\xi,\eta)$ and $b_j(\xi,\eta) \in C(G)$, j = 1, 2, 3, 4, is reduced to the normal form (2).

We now consider the system (2) in some special cases.

In the case where $p = y^k$ and $q = 0$ (k being an arbitrary positive constant) a (p, q)-wave function is called a y^k-*wave* function. If in addition. $k = 0$ then it is called a wave function. Note that for y^k-wave functions the system (2) is of the form

$$y^k \frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}, \quad y^k \frac{\partial U}{\partial y} = \frac{\partial V}{\partial x}. \tag{3}$$

Observe that (3) implies

$$\frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial y^2} - \frac{k}{y}\frac{\partial U}{\partial y} = 0,$$

$$\frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 V}{\partial y^2} + \frac{k}{y}\frac{\partial V}{\partial y} = 0. \tag{4}$$

if $U(x, y), V(x, y) \in C(G)$. It is well known that some problems arising in the theory of oscillations of an elastic solid and wave propagations can be reduced to those of finding solutions of the equation (4).

2.1. Integral representation

Let $f(z) = u(x,y) + iv(x,y)$ be a wave function in G, C_1 and C_2 be real constants. We shall prove the following

THEOREM 1. *The function $F(z) = U(x, y) + i\,V(x,y)$ defined by the following formula*

$$y^k [U(x, y) - C_1] - i\,[V(x, y) - C_2] =$$

$$= \int_0^y (y-\gamma)(y^2-\gamma^2)^{\frac{k}{2}-1} f(x, \gamma)\, d\gamma +$$

$$+ \int_0^y (y+\gamma)(y^2-\gamma^2)^{\frac{k}{2}-1} \overline{f(x, \gamma)}\, d\gamma. \tag{5}$$

is a y^k-wave function in G if G contains an entire line segment joining two arbitrary points of G with the same abscissa and if one of the following conditions is satisfied: a, G lies in the upper half-plane and the boundary of G contains a segment L of the real axis such that $v(x,y)_{/L} = 0$. b) The region G is symmetric about the real axis ($y = 0$) and $v(x, y)\,|_{\overline{G} \cap (y=0)} = 0$.

Proof. We first observe that $F(z) = U(x,y) + iV(x, y)$ is a y^k-wave function in G if and only if the function $\varphi(x, y)$ defined by

$$\varphi(x, y) = y^{\frac{k}{2}} [U(x, y) - C_1] - i\, y^{-\frac{k}{2}} [V(x, y) - C_2], \tag{6}$$

satisfies the equation

$$\frac{\partial \varphi}{\partial x} + i\,\frac{\partial \overline{\varphi}}{\partial y} = i\,\frac{k}{2y}\,\varphi(x, y),\ (x, y) \in G. \tag{7}$$

Hence, to prove the theorem it is enough to find two real functions M (y,γ) and N(y, γ) such that

$$\varphi(x, y) = \int_0^y M(y, \gamma) f(x, \gamma)\, d\gamma + \int_0^y N(y, \gamma) \overline{f(x, \gamma)}\, d\gamma \tag{8}$$

satisfies (7).

Since $f(x,y)$ is a wave function, we get

$$\frac{\partial f(x,y)}{\partial x} = i \frac{\partial \overline{f(x,y)}}{\partial y}.$$

This equality together with (8) shows that $\varphi(x,y)$ satisfies (7) if

$$\frac{\partial M}{\partial y} - \frac{\partial M}{\partial \gamma} = \frac{k}{2y} N,$$
$$\frac{\partial N}{\partial y} + \frac{\partial N}{\partial \gamma} = \frac{k}{2y} M, \tag{9}$$

and

$$M(y,y) = 0.$$

Hence, $M(y, \gamma)$ and $N(y,\gamma)$ are respectively the solutions of the following equations:

$$\frac{\partial^2 M}{\partial y^2} - \frac{\partial^2 M}{\partial \gamma^2} + y^{-1}\left(\frac{\partial M}{\partial y} - \frac{\partial M}{\partial \gamma}\right) - \frac{k^2 y^{-2}}{4} M = 0,$$
$$\frac{\partial^2 N}{\partial y^2} - \frac{\partial^2 N}{\partial \gamma^2} + y^{-1}\left(\frac{\partial N}{\partial y} + \frac{\partial N}{\partial \gamma}\right) - \frac{k^2 y^{-2}}{4} N = 0.$$

Taking account of the condition (9) we find

$$M(y, \gamma) = y^{-\frac{k}{2}} (y - \gamma) (y^2 - \gamma^2)^{\frac{k}{2} - 1},$$
$$N(y, \gamma) = y^{-\frac{k}{1}} (y + \gamma) (y^2 - \gamma^2)^{\frac{k}{2} - 1} \tag{10}$$

Thus, the function (8) where $M(y, \gamma)$ and $N(y,\gamma)$ are given by the last formulas is a solution of (7). This completes the proof of the Theorem 1.

Suppose now that G is an unbounded region and the wave function f(z) satisfies the following condition: $f(z) = 0(|z|^{-k-\varepsilon})$ for $z \to \infty$, where ε is an arbitrary positive constant.

Then, arguing as above we obtain

THEOREM 2. *The function* $F(z) = U(x, y) + iV(x, y)$ *given by the formula*

$$F(z) = \int_\infty^y \Big\{ y^{1-k} \Big[u(x, \gamma) \cos\Big(\frac{k}{2} - 1\Big)\pi + v(x, \gamma) \sin\Big(\frac{k}{2} - 1\Big)\pi \Big] +$$

$$+ i\gamma \Big[u(x, \gamma) \sin\Big(\frac{k}{2} - 1\Big)\pi + v(x, \gamma) \cos\Big(\frac{k}{2} - 1\Big)\pi \Big] \Big\} \left(\gamma^2 - y^2\right)^{\frac{k}{2} - 1} d\gamma \tag{11}$$

is a y^k*-wave function in G.*

2.2. Inversion formula

Using the inverse transformation for an integral equation of the Abel type we can write the inversion formulas for the representation (5) and (11).

THEOREM 3. *Let a y^k-wave function $F(z) = U(x, y) + iV(x, y)$ be expressed in terms of a wave function $f(z) = u(x, y) + iv(x, y)$ by (5). Then, conversely, $f(z)$ can be represented as follows*

$u(x, y) + yv(x, iy) =$

$$= \begin{cases} K \dfrac{\partial}{\partial y} \displaystyle\int_0^y \dfrac{\partial^m \{\gamma^{k-1}[U(x,\gamma) - C_1] + i[V(x,\gamma) - C_2]\}}{(\partial \gamma^2)^m} \dfrac{\gamma d\gamma}{(y^2 - \gamma^2)^{\frac{k}{2} - m}}, & k \neq 2m, \\ Ky \dfrac{\partial^m \{y^{k-1}[U(x,y) - C_1] + i[V(x,y) - C_2]\}}{(\partial y^2)^m}, & k = 2m, \end{cases} \tag{12}$$

where $K = 2\left[\Gamma\left(\frac{k}{2}\right)\Gamma\left(m - \frac{k}{2} + 1\right)\right]^{-1}$, *m is the integer part of the number* $\frac{k}{2}$: $m = \left[\frac{k}{2}\right]$.

Note that from the Theorems 1 and 3 it follows that the integral representation (5) provides a one-to-one correspondence between a y^k-wave function and a wave function.

3. INTEGRAL REORESENTATION OF (Y^k, Y^k)—WAVE FUNCTIONS ANB INAERSION FORMULA

We now consider the system (2) when $p = q = y^k$. In this case, if $U(x, y)$ and $V(x, y) \in C^2(G)$ then

$$\frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial y^2} + \frac{k}{y}\frac{\partial U}{\partial x} - \frac{k}{y}\frac{\partial U}{\partial y} = 0,$$

$$\frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 V}{\partial y^2} + \frac{k}{y}\frac{\partial V}{\partial x} + \frac{k}{y}\frac{\partial V}{\partial y} = 0.$$

3.1. Integral representation

Let the region G and the wave function $f(z) = u(x, y) + iv(x, y)$ be defined as in the previous section. We suppose also that
$u(x, y)|_{\bar{G} \cap (y=0)} = 0$.

THEOREM 4. *The function $F(z) = U(x, y) + iV(x, y)$ defined by the formula*

$$y^k(1 - i)U(x, y) - iV(x, y) =$$
$$= \int_0^y (y - \gamma)^{k-1} [i + (y - \gamma)(y + \gamma)^{-1}] f(x, \gamma) d\gamma +$$
$$+ \int_0^y (y - \gamma)^{k-1} [1 - i(y - \gamma)(y + \gamma)^{-1}] \overline{f(x, \gamma)}\, d\gamma, \tag{13}$$

is a (y^k, y^k) — wave function in G:

Proof. We shall use the same reasoning as in the proof of Theorem 1. Indeed, it is easy to see that $F(z) = U(x, y) + iV(x, y)$ is $a(y^k, y^k)$-wave function in G if and only if the function $H(x, y)$ given by the formula

$$H(x, y) = y^{\frac{k}{2}} (1 - i)\, U(x, y) - iy^{-\frac{k}{2}} V(x, y), \tag{14}$$

satisfies the following equation

$$\frac{\partial H}{\partial x} + i\, \frac{\partial \overline{H}}{\partial y} = \frac{k}{2y} (i - 1)\, H - \frac{k}{2y}\, \overline{H},\ (x, y) \in G. \tag{15}$$

Hence to prove the theorem, it suffices to find $M(y, \gamma)$ and $N(y, \gamma)$ such that the function

$$H(x, y) = \int_0^y M(y, \gamma) f(x, \gamma) d\gamma + \int_0^y N(y, \gamma) \overline{f(x, \gamma)}\, d\gamma \tag{16}$$

satisfies (15).

For this purpose, we can take $M(y, \gamma)$ and $N(y, \gamma)$ as the solution of the equations

$$\frac{\partial M}{\partial \gamma} - \frac{\partial \overline{M}}{\partial y} = - \frac{k}{2y} (1 + i)N - i\frac{k}{2y}\, \overline{M},$$

$$\frac{\partial N}{\partial \gamma} + \frac{\partial \overline{N}}{\partial y} = \frac{k}{2y} (1 + i)M + i\, \frac{k}{2y}\overline{N}, \tag{17}$$

satisfying the conditions

$$ReM(y, \gamma)|_{\gamma = y} = 0; \tag{18}$$

$$ImN(y, \gamma)|_{\gamma = y} = 0. \tag{19}$$

Consequently, the function $N(y, \gamma)$ can be chosen as a solution of the following second-order equation

$$\frac{\partial^2 N}{6\gamma^2} - i\, \frac{\partial^2 N}{\partial y^2} + (1 - i)\, \frac{\partial^2 N}{\partial y \partial \gamma} - \frac{1}{2y} (k + 2i + ki)\, \frac{\partial \overline{N}}{\partial \gamma} -$$

$$- i\, \frac{1}{y}\, \frac{\partial N}{6y} + i\, \frac{k^2}{4y^2} N = 0$$

satisfying (19)

It is easy to verify that this solution is given by

$$N(y, \gamma) = y^{-\frac{k}{2}} (y - \gamma)^{k-1} [1 - i(y - \gamma)(y + \gamma)^{-1}] \tag{20}$$

Similarly, the function

$$M(y, \gamma) = y^{-\frac{k}{2}} (y - \gamma)^{k-1} [(y - \gamma)(y + \gamma)^{-1} + i] \tag{21}$$

is the solution of system (17) satisfying (18).

The proof of the theorem is thus complete.

3. 2. Inversion formula

Applying the inverse transformation for an integral equation of the Abel type we can obtain the inversion formula for (13).

THEOREM 5. *Let a* (y^k, y^k)*-wave function* $F(z) = U(x, y) + iV(x, y)$ *be expressed in terms of a wave function* $f(z) = u(x, y) + iv(x, y)$ *by* (13). *Then the inversion formula for this representation is given as follows*
$u(x, y) - v(x, y) =$

$$= \begin{cases} \dfrac{1}{\Gamma(k)\Gamma(m-k+1)} \dfrac{\partial}{\partial y} \displaystyle\int_0^y \dfrac{\partial^m V(x, \gamma)}{(\partial\gamma)^m} \dfrac{d\gamma}{(y-\gamma)^{k-m}}, & k \neq m, \\ \dfrac{1}{(k-1)!} \dfrac{\partial^k V(x, y)}{(\partial y)^k}, & k = m, \end{cases} \tag{22}$$

where $m = [k]$.

On account of (2) and (22) we see that

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y} =$$

$$= \begin{cases} \dfrac{1}{\Gamma(k)\Gamma(m-k+1)} \dfrac{\partial}{\partial y} \displaystyle\int_0^y \dfrac{\partial^m}{\partial\gamma^m} \left[\gamma^k \left(\dfrac{\partial U}{\partial x} - \dfrac{\partial U}{\partial \gamma} \right) \right] \dfrac{d\gamma}{(y-\gamma)^{k-m}}, & k \neq m, \\ \dfrac{1}{(k-1)!} \dfrac{\partial^k}{\partial y^k} \left[y^k \left(\dfrac{\partial U}{\partial x} - \dfrac{\partial U}{\partial y} \right) \right], & k = m. \end{cases} \tag{23}$$

4. APPLICATION

In this section using the obtained results we can find explicitly solutions of some boundary value problems for the (p, q)-wave functions.

PROBLEM 1. Let G_1 be the first orthant $\{(x, y);\ x > 0,\ y > 0\}$.

Find a y^k-wave function $F(z)$ in G_1 such that

$$U(0, y) = H(y)\ (0 \leqslant y < \infty), \tag{24}$$

$$U(x, 0) = G(x)\ (0 \leqslant x < \infty), \tag{25}$$

where $H(y) \in C^3\ (y \geqslant 0)$, $G(x) \in C^3(x \geqslant 0)$, $G(0) = H(0)$.

We shall find the solution $F(z)$ of Problem 1 in the form (5) such that the real and imaginary parts $u(x,y)$ and $v(x, y)$ of the wave function $f(z)$ are real wave functions in G_1 and

$$v(x, 0) = 0\ (0 \leqslant x < \infty). \tag{26}$$

For simplicity of presentation, et us suppose that $k = 2$. Applying the inversion formula (12) we have from (24) and (25)

$$u(0, y) = \frac{K}{2} \frac{d}{dy} [y(H(y) - C_1)] \equiv h(y)\ (0 \leqslant y < \infty), \tag{27}$$

$$u(x,0) = \frac{K}{2}[G(x) - C_1] \equiv g(x) \; (0 \leqslant x < \infty), \tag{28}$$

where C_1 is an arbitrary real constant.

It follows from (26), (27) and (28) that

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0 \; (0 \leqslant x < \infty). \tag{29}$$

$$\left.\frac{\partial v}{\partial y}\right|_{y=0} = g'(x) \; (0 \leqslant x < \infty) \tag{30}$$

$$\left.\frac{\partial v}{\partial x}\right|_{x=0} = h'(y) \; (0 \leqslant y < \infty). \tag{31}$$

Since $u(x,y)$ and $v(x,y)$ are real wave functions we have

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial x^2}, \tag{32}$$

$$\frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial x^2}. \tag{33}$$

It is known [1] that the solutions of the problems (32), (27), (28), (29) and (33), (26), (30), (31) are given by

$$u(x, y) = \begin{cases} \frac{1}{2}[g(x+y) + g(x-y)] \; (0 \leqslant y \leqslant x < \infty), \\ h(y-x) + \frac{1}{2}[g(y+x) - g(y-x)] \; (0 \leqslant x < y < \infty), \end{cases} \tag{34}$$

$$v(x, y) = \begin{cases} \frac{1}{2}[g(x+y) - g(x-y)] \; (0 \leqslant y \leqslant x < \infty), \\ -h(y-x) + \frac{1}{2}[g(y+x) + g(y-x)] + h(0) - g(0) \\ \qquad (0 \leqslant x < y < \infty). \end{cases} \tag{35}$$

The function $F(z)$ defined by (5), (34) and (35) yields the desired solution of Problem 1.

PROBLEM 2. Let G_2 be the half-strip $\{(x, y): 0 < x < l, \quad 0 < y < \infty\}$. Find a y^k-wave function $F(z)$ in G_2 such that

$$U(0, y) = H(y),$$

$$U(l, y) = R(y) \quad (0 < y < \infty), \tag{36}$$

$$U(x, 0) = G(x) \quad (0 \leqslant x \leqslant l,) \tag{37}$$

where $H(y)$ and $R(y) \in C^3$ $(y \geqslant 0)$, $G(x) \in C^3$ $(0 \leqslant x \leqslant l)$, $G(0) = H(0)$.

We shall consider only the case $k = 2$ and find the required solution in the form (5). Then in view of (12), (36) and (37) we have

$$u(0, y) = \frac{K}{2}\frac{d}{dy}[y(H(y) - C_1)] \equiv h(y),$$

$$u(l, y) = \frac{K}{2}\frac{d}{dy}[y(R(y) - C_1)] \equiv r(y) \quad (0 \leqslant y < \infty), \tag{38}$$

$$u(x, 0) = \frac{K}{2}[G(x) - C_1] \equiv g(x) \quad (0 \leqslant x \leqslant l). \tag{39}$$

Let us set

$$\widehat{h}(y) = \begin{cases} 0 & (y < 0), \\ h(y) & (y \geqslant 0), \end{cases} \qquad \widehat{r}(y) = \begin{cases} 0 & (y < 0), \\ r(y) & (y \geqslant 0), \end{cases}$$

$$\tilde{g}(x) = \begin{cases} g(x) & (0 \leqslant x \leqslant l), \\ -g(-x) & (-l \leqslant x \leqslant 0), \end{cases}$$

where the functions $h(y)$, $r(y)$ and $g(x)$ are given by (38) and (39) with $C_1 = G(0)$. Denote by $g(x)$ the $2\,l$-periodic function which coincides with $g(x)$ over the interval $[-l, l]$.

It is known [1] that the problem (32), (29), (38) and (39) has the solution

$$u(x, y) = \frac{1}{2} [\check{g}(x + y) + \check{g}(x - y)] +$$

$$+ \sum_{n=o}^{\infty} [\widehat{h}(y - x\ 2nl) - \widehat{h}(y + x - 2(n + 1)l)] +$$

$$+ \sum_{n=o}^{\infty} [\widehat{r}(y + x - (2n + 1)l) - \widehat{r}(y - x - (2n + 1)l)]$$

$$(0 \leqslant x \leqslant l,\ 0 \leqslant y < \infty). \tag{40}$$

To find t e real wave function $v(x, y)$, observe by virtue of (38) and (39) that $v(x,y)$ u t satisfy the boundary conditions

$$\left.\frac{\partial v}{\partial x}\right|_{x=0} = h'(y),$$

$$\left.\frac{\partial}{\partial x}\right|_{x=l} = r'(y) \quad (0 \leqslant y < \infty), \tag{41}$$

$$\left.\frac{\partial v}{\partial y}\right|_{y=0} = g'(x) \quad (0 \leqslant x \leqslant l). \tag{42}$$

The problem (33), (26), (41) and (42) admits the solution

$$v(x, y) = \frac{1}{2} [\check{g}(x + y) - \widehat{g}(x - y)] -$$

$$- \sum_{n=0}^{\infty} [\widehat{h}(y - x\quad 2\ l) + \widehat{\ }(y + x - 2(n + 1)l)] +$$

$$+ \sum_{n=0}^{\infty} [\quad y + \ - 2\ + 1)l) + \widehat{r}(y - \ - 2\ + 1\,l\,]$$

$$0 \leqslant x \leqslant l, \quad 0 \leqslant x < \infty). \tag{43}$$

Thus the solution of Problem 2 is given by (5), (40) and (43).

PROBLEM 3. Let G_1 be defined as in Problem 1. Find a (y^k, y^k) — wave function $F(z)$, in G_1 such tha

$$V(0, y) = M(y)\ (0 \leqslant y < \infty); \tag{44}$$

where $M(y) \in C^{k+2}(y \geqslant 0)$, $M^{(k)}(y)\Big|_{y=0} = 0$.

By (13) we fin the solution in the form

$$U(x, y) = y^{-k+1}\int_0^y (y-\gamma)^{k-1}(y+\gamma)^{-1}[v(x,\gamma)-u(x,\gamma)]d\gamma,$$

$$V(x, y) = -\int_0^y (y-\gamma)^{k-1}[v(x,\gamma)-u(x,\gamma)]d\gamma, \tag{45}$$

where $u(x, y)$, $v(x, y) \in C^2(G)$ and

$$u(x, 0) = v(x, 0) = 0 \ (0 \leqslant x < \infty). \tag{46}$$

Without loss of generality we assume that k is an integer.

From (22), (44) and (46) it follows that

$$[\varphi(x,y) \equiv u(x,y) - v(x,y)]\Big|_{x=0} = \frac{1}{(k-1)!}M^{(k)}(y) \equiv m(y) \ (0 < y < \infty), \tag{47}$$

$$\varphi(x, 0) = 0, \ \frac{\partial\varphi}{\partial y}\Big|_{y=0} = 0 \ (0 \leqslant x < \infty). \tag{48}$$

By assumption

$$\frac{\partial^2\varphi}{\partial y^2} = \frac{\partial^2\varphi}{\partial x^2}. \tag{49}$$

The solution of the problem (49), (47) and (48) can be written in the form

$$\varphi(x,y) = u(x,y) - v(x,y) = \begin{cases} 0 & (0 \leqslant y \leqslant x < \infty), \\ m(y-x) & (0 \leqslant x < y < \infty) \end{cases} \tag{50}$$

Combining (50) and (45) yields the desired solution,

PROBLEM 4. Let G_2 be defined as in Problem 2. Find a(y^k, y^k) — wave function $F(z)$ in G such that

$$V(0,y) = M(y) \ (0 \leqslant y < \infty),$$
$$V(l,y) = N(y) \ (0 \leqslant y < \infty), \tag{51}$$

where

$$M(y),\ N(y) \in C^{k+2}\ (y \geqslant 0),\ M^{(k)}(y)\Big|_{y=0} = N^{(k)}(y)\Big|_{y=0} = 0.$$

It can be verified that the desired solution has the form (45) where

$$u(x,y) - v(x,y) = \sum_{i=0}^{\infty}[\widehat{m}(y-x-2jl) - \widehat{n}(y-x-(2j+1)l)]$$
$$(0 \leqslant x \leqslant l,\ 0 \leqslant y < \infty), \tag{52}$$

$$\widehat{m}(y) = \begin{cases} 0 (y < 0), \\ m(y)\ (y \geqslant 0), \end{cases} \quad \widehat{n}(y) = \begin{cases} 0(y < 0) \\ n(y)\ (y \geqslant 0), \end{cases}$$

$$m(y) = \frac{1}{(k-1)!}M^{(k)}(y),\ n(y) = \frac{1}{(k-1)!}N^{(k)}(y)\ (0 \leqslant y < \infty),$$

$$n(y) = 0 (0 \leqslant y \leqslant l).$$

k is an arbitrary integer.

For the above problem to be solvable the functions $\widehat{m}(y)$ and $\widehat{n}(y)$ must satisfy the following condition.

$$\widehat{m}(y) = \widehat{n}(y+l)\,(0 \leqslant y < \infty).$$

Finally, it is worth noticing that with the help of the obtained integral representations we can also solve other boundary value problems involving bounded regions.

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