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ON REPRESENTING SYSTEMS FOR FUNCTIONS OF SEVERAL VARIABLES

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§I. INTRODUCTION

1.1. During the last two decades the problems of representation of analytic functions by means of series of exponentials or by more general functional series have been extensively investigated (see, e.g., [1], [2]). This interest stems from the fact that these representations allow us to study rather deep functional properties of analytic functions and, furthermore, have important applications to functional equations. Results in this subject were obtained first by Leont'ev [1] and later on, by Korobeinik and his school, who developed systematically a theory of representing systems [2].

It should be noted that at present, while the theory of representing systems for analytic functions of one complex variable has reached a sufficiently mature stage of development, the theory for several complex variables is only in its initial stage. In this field many open problems still call for investigations.

In a previous work [3] we have proved that the notion of absolute representing systems is invariant with respect to projective topological tensor product in Fréchet spaces. This result has allowed us to obtain a useful criterion of absolute representation for a multidimensional system of analytic functions in polycylindrical domains.

In the present paper, using the same idea of invariantness we shall establish criteria of absolute representation for multi-dimensional systems in some spaces of entire and analytic functions of several complex variables. We shall also discuss properties of inward-continuability of representing systems of exponentials in convex domains of several variables.

1.2. Let us recall some definitions and notations which will be used throughout this paper.

$[\rho,\sigma]$ $(\rho > 0,\ 0 \leqslant \sigma < +\infty)$ will denote the class of all entire functions $y(z)$ on $\mathbf{C}$ such that

$$\varlimsup_{r\to\infty} r^{-\rho} \ln \mathcal{M}(r, y) \leqslant \delta,$$

where $\mathcal{M}(r, y) = \sup \{ \mid y(z) \mid : \mid z \mid = r\}$;

$[\rho, \infty]$ $(\rho > 0)$ is the class of all entire functions on $\mathbf{C}$ of order at most ρ;

$V(W)$ (W being a domain in $\mathbf{C}^n$) is the space of analytic functions in W, with the topology of uniform convergence on compacta;

$V(\mathcal{K})$ ($\mathcal{K}$ being a compact set in $\mathbf{C}^n$) is the space of germs of functions analytic on compact set, with the topology of inductive limit: $V(\mathcal{K}) = \lim \operatorname{ind} V(\omega)$, ω is a neighbourhood of $\mathcal{K}$. $\omega \supset \mathcal{K}$

If $z, \xi \in \mathbf{C}^n$ then we denote $\mid z \mid = (z_1 \bar{z}_1 + \dots + z_n \bar{z}_n)^{1/2}$; $\| z \| = \mid z_1 \mid + \dots + \mid z_n \mid$; $< z, \xi > = z_1 \xi_1 + \dots + z_n \xi_n$.

The support function of a convex set G in $\mathbf{C}^n$ $(n \geqslant 1)$ is defined as follows:

$$\text{for } n = 1: h(-\varphi) = \sup_{z\in G} \operatorname{Re}(z e^{-i\varphi}),\ 0 \leqslant \varphi \leqslant 2\pi;$$

$$\text{for } n \geqslant 2: h_G(\xi) = \sup_{z\in G} \operatorname{Re} < z, \xi >,\ \xi \in C^n.$$

A sequence $\{x_K\}$ of nonzero elements of a locally convex space H is said to be a *representing system* (an *absolute representing system*) in H or, briefly, RS (respectively, ARS) in H if any element x of H can be represented as the sum of a convergent (respectively, absolute convergent) series in H:

$$x = \sum_{k=1}^{\infty} c_k x_k$$

§ 2. CRITERIA OF ABSOLUTE REPRESENTATION FOR MULTI-DIMENSIONAL SYSTEMS

2. 1. Let $\rho > 0$, $0 \leqslant \sigma < +\infty$. As is wellknown, $[\rho, \sigma]$ and $[\rho, \infty]$ are nuclear Fréchet spaces with bases $\{z^k\}_{k=0}^{\infty}$ whose topologies are defined, respectively, by the countable sets of norms

$$\sup_{\mathbf{C}} \left\{ \mid y(z) \mid . \exp [-(\sigma + \varepsilon_m) \mid z \mid^{\rho}] \right\} < +\infty,$$

$$\sup_{\mathbf{C}} \left\{ \mid y(z) \mid . \exp \left[- \mid z \mid^{\rho+\varepsilon_m} \right] \right\} < +\infty,$$

where $\varepsilon_m \downarrow 0$ $(m = 1, 2, \dots)$.

Now let n be a natural number and p, q, r be non-negative integer numbers such that $p + q + r = n$. Let

$$\mu(z, \varepsilon) \stackrel{df}{=} \varepsilon \sum_{i=1}^{p} | z_i |^{\rho_i} + \sum_{j=p+1}^{p+q} (\sigma_j + \varepsilon) | z_j |^{\rho_j} + \sum_{s=p+q+1}^{n} | z_s |^{\rho_s} + \varepsilon ,$$

where $\varepsilon > 0, 0 < \sigma_j < \infty$ $(j = p + 1, \ldots, p + q)$, $\rho_k > 0$ $(k = 1, 2, \ldots, n)$.

We introduce a Fréchet space $H \stackrel{df}{=} \lim_{m} pr\, H_m$ where

$$H_m = \left\{ f(z) \in \mathcal{H} (\mathbf{C}^n) : \sup_{\mathbf{C}^n} \frac{| f(z) |}{\exp \mu (z, \varepsilon_m)} = \| f \|_m < + \infty \right\}$$

is a Banach space with a norm $\| \cdot \|_m$, $\varepsilon_m \downarrow 0$ $(m = 1, 2, \ldots)$.

We note the elementary formula:

$$\sup_{t>0} t^k e^{-\mu t^\nu} = \left(\inf_{t>0} e^{\mu t^\nu} t^{-k} \right)^{-1} = \left(\frac{k}{e\mu\nu}\right)^{\frac{k}{\nu}}$$

$$\mu > 0, \nu > 0, k \geqslant 0 \tag{1}$$

LEMMA 1. *The monomials* $\{ z^k \}_{\|k\|=0}^{\infty}$ *form an absolute basis in the Fréchet space* **H**.

Proof. Take an arbitrary fuction f from H. It can be represented uniquely by a Taylor's series about $z = 0$:

$$f(z) = \sum_{\|k\|=0}^{\infty} a_k z^k ,$$

where $k = (k_1, \ldots, k_n)$, $k_j \geqslant 0$ $(j = 1, \ldots, n)$, $z^k = z_1^{k_1} \ldots z_n^{k_n}$.

Fix an arbitrary natural number $m > 1$. Then

$$\left\| f - \sum_{\|k\| \leqslant N} a_k z^k \right\|_m = \left\| \sum_{\|k\|>N} a_k z^k \right\|_m \leqslant \sum_{\|k\|>N} | a_k | \cdot \| z^k \|_m .$$

We will show that the series $\sum_{\|k\|=0}^{\infty} |a_k| \cdot \| z^k \|_m$ converges. Indeed, using (1) we have

$$\| z^k \|_m = \prod_{i=1}^{p} \left(\frac{k_i}{e\, \varepsilon_m \rho_i}\right)^{k_i / \rho_i} \prod_{j=p+1}^{p+q} \left(\frac{k_j}{e(\sigma_j + \varepsilon_m) \rho_j}\right)^{k_j / \rho_j} \times$$

$$\times \prod_{s=p+q+1}^{n} \left(\frac{k_s}{e(\rho_s + \varepsilon_m)}\right)^{k_s / \rho_s + \varepsilon_m} \tag{2}$$

Furthermore, for every $\varepsilon > 0$ we can write

$$\sup_{t \in \mathbf{R}_+^n} \frac{\mathcal{M}(f, t)}{\exp \mu(t, \varepsilon)} \leqslant \sup_{z \in \mathbf{C}^n} \frac{|f(z)|}{\exp \mu(z, \varepsilon)} = \|f\|_{\varepsilon},$$

where $t = (t_1, ..., t_n) \in \mathbf{R}_+^n$, $\mathcal{M}(f, t) = \sup\{|f(z)| : |z_j| = t_j, j = 1, ..., n\}$. Therefore, for every $t \in \mathbf{R}_+^n$: $\mathcal{M}(f, t) \leqslant \|f\| . \exp \mu(t, \varepsilon)$, and by Cauchy's inequality for the coefficients a_k we get

$|a_k| \leqslant \|f\|_{\varepsilon} \cdot \dfrac{\exp \mu(t, \varepsilon)}{t^k}$ $\forall\, t_j > 0$, $\forall\, k_j \geqslant 0$ $(j = 1, ..., n)$. Hence by (1),

$$|a_k| \leqslant \|f\|_{\varepsilon} \cdot \prod_{i=1}^{p} \left(\frac{e \rho_i \varepsilon}{k_i}\right)^{k_i/\rho_i} . \prod_{j=p+1}^{p+q} \left(\frac{e(б_j + \varepsilon)\rho_j}{k_j}\right)^{k_j/\rho_j} \times$$

$$\times \prod_{s=p+q+1}^{n} \left(\frac{e(\rho_s + \varepsilon)}{k_s}\right)^{k_s/\rho_s + \varepsilon} \tag{3}$$

Combining (2) and (3) yields

$$|a_k| \cdot \|z^k\|_m \leqslant \|f\|_{\varepsilon} . \prod_{i=1}^{p} \left(\frac{\varepsilon}{\varepsilon_m}\right)^{k_i/\rho_i} . \prod_{j=p+1}^{p+q} \left(\frac{б_j + \varepsilon}{б_j + \varepsilon_m}\right)^{k_j/\rho_j} \times$$

$$\times \prod_{s=q+p+1}^{n} \left(\frac{k_s}{e(\rho_s + \varepsilon_m)}\right)^{k_s/\rho_s + \varepsilon_m} \left(\frac{e(\rho_s + \varepsilon)}{k_s}\right)^{k_s/\rho_s + \varepsilon}$$

It is clear that the series $\sum\limits_{k_i=0}^{\infty} \left(\dfrac{\varepsilon}{\varepsilon_m}\right)^{k_i/\rho_i}$ $(i = 1, ..., p)$

and $\sum\limits_{k_j=0}^{\infty} \left(\dfrac{б_j + \varepsilon}{б_j + \varepsilon m}\right)^{k_j/s_j}$ $(j = p + 1, ..., p + q)$ converge for $\varepsilon < \varepsilon_m$.

Moreover, if $\rho > 0$, then $\dfrac{k}{e(\rho + \varepsilon_m)} > 1$ for all $k > k_0$. Consequently, since $\varepsilon < \varepsilon_m$, we can write

$$\left(\frac{k}{e(\rho + \varepsilon_m)}\right)^{k/\rho + \varepsilon_m} . \left(\frac{e(\rho + \varepsilon)}{k}\right)^{k/\rho + \varepsilon} <$$

$$< \left(\frac{k}{e(\rho + \varepsilon_m)}\right)^{k/\rho + \varepsilon} \cdot \left(\frac{e(\rho + \varepsilon)}{k}\right)^{k/\rho + \varepsilon} = \left(\frac{\rho + \varepsilon}{\rho + \varepsilon_m}\right)^{k/\rho + \varepsilon}$$

Hence, the series

$$\sum_{k_s=0}^{\infty} \left(\frac{k_s}{e(\rho_s + \varepsilon_m)}\right)^{k_s/\rho_s + \varepsilon_m} \left(\frac{e(\rho_s + \varepsilon)}{k_s}\right)^{ks/\rho_s + \varepsilon}$$

$(s = p + q + 1, ..., n)$ converge too.

It follows that the series $\sum_{\|k\|=0}^{\infty} |a_k| \cdot \| z^k \|_m$ converges. That is

$\| f - \sum_{\|k\| \leqslant N} a_k z^k \|_m \to 0$ as $N \to \infty$ $(m = 1, 2, \ldots)$ Q.E.D.

Remark. When two of the numbers p, q, r are equal to zero, the space H was already considered by Rolewicz [4].

Now let $V_i = [\rho_i, 0]$ $(i = 1, \ldots, p)$, $V_j = [\rho_j, \sigma_j]$ $(j = p + 1, \ldots, p + q)$, V_S $[\rho_S, \infty]$ $(s = p + q + 1, \ldots, n)$ with the sets of norms $\| \cdot \|_m^i$, $\| \cdot \|_m^j$, $\| \cdot \|_m^s$ $(m = 1, 2, \ldots)$ respectively.

It is clear that if $f(z) \in H$, then $f \in V_l$ with respect to the variable z_l when the values of the remaining variables are fixed, $l = 1, \ldots, n$. Further, if

$$\Phi_l \left\{ \varphi_k^{(l)}(z_l) \right\}_{k=1}^{\infty} \subset V_l \ (l = 1, \ldots, n),$$

it is easy to see that

$$\prod_{l=1}^{n} \Phi_l \overset{df}{=} \left\{ \varphi_{k_1}^{(1)}(z_1) \ldots \varphi_{k_n}^{(n)}(z_n) \right\}_{k_1, \ldots, k_n = 1}^{\infty} \subset H.$$

As in the case of analytic functions in domains (see [3]), it is not hard to verify that if $\prod_{l=1}^{n} \Phi_l$ is ARS in H, then each component Φ_l is ARS in V_l $(l = 1, \ldots, n)$. Moreover, $\forall m \geqslant 1$

$$\| z^k \|_m = \sup_{\mathbf{C}^n} \frac{|z^k|}{\exp \mu(z, \varepsilon_m)} = \prod_{l=1}^{n} \| z_l^{k_l} \|_m^l, \quad \|k\| = 0, 1, 2, \ldots,$$

On the other hand, for $\widehat{r}_m = \widehat{\bigotimes}_{1 \leqslant l \leqslant n} \| \cdot \|_m^l$, we have

$$\widehat{r}_m(z_1^{k_1} \otimes \ldots \otimes z_n^{k_n}) = r_m(z_1^{k_1} \otimes \ldots \otimes z_n^{k_n}) =$$

$$= \| z_1^{k_1} \|_m^1 \ldots \| z_n^{k_n} \|_m^n \ (m = 1, 2, \ldots; \|k\| = 0, 1, 2, \ldots).$$

Setting

$$L\left(\sum_{\|k\|=0}^{\infty} c_k z^k\right) = \sum_{\|k\|=0}^{\infty} c_k z_1^{k_1} \otimes \ldots \otimes z_n^{k_n},$$

it can easily be verified in the same way as in the case of analytic functions, that L is an isomorphism of H onto $\widehat{\bigotimes}_{1 \leqslant l \leqslant n} V_l$.

We thus obtain the following result.

THEOREM 1. *Let* $\Phi_l = \{ \varphi_k^{(l)}(z_l) \}_{k=1}^{\infty} \subset V_l$ $(1 \leqslant l \leqslant n)$. *The system* $\prod_{l=1}^{n} \Phi_l$ *is ARS in H if and only if each component* Φ_l *is ARS in* V_l, $1 \leqslant l \leqslant n$.

2. 2. Now let G_l be a domain in the w_l -plane such that there exists a basis $\{ x_k^{(l)}(w_l) \}_{k=0}^{\infty}$ in $V(G_l)$, $1 \leqslant l \leqslant N$ (this will be the case if, for example G_l is a finitely connected domain in the plane of the complex variable w_l). Denote by $(HG)_{n,N}$ the space of all functions $f(z, w) \in V(\mathbf{C}^n \times G)$, $z = (z_1, \ldots, z_n)$, $w = (w_1, \ldots, w_N)$, $G = G_1 \times \ldots \times G_N$, which satisfy the following conditions:

a) $f(z, w) \in H$ with respect to z when w is fixed;

b) $f(z, w) \in V(G)$ with respect to w when z is fixed;

c) $\sup \left\{ \left| f(z, w) \right| . \exp \left[- \mu(z, \varepsilon_\nu) \right] : z \in \mathbf{C}^n, w \in K_\nu \right\} =$

$= \| f \|_\nu < +\infty$, where $K_\nu = \overline{K}_\nu \uparrow G$, $\varepsilon_\nu \downarrow 0$ $(\nu = 1, 2, \ldots)$.

LEMMA 2. *The system*

$$\left\{ z_1^{k_1} \ldots z_n^{k_n} x_{m_1}^{(1)}(w_1) \ldots x_{m_N}^{(N)}(w_N) \right\}_{k_1, \ldots, k_n, m_1, \ldots, m_N = 0}^{\infty}$$

forms an absolute basis in the Fréchet space $(HG)_{n,N}$.

Proof. The proof is similar to that of Lemma 1.

Let us expand the function $f(z, w)$ from $(HG)_{n,N}$ in a series with respect to the basis $\{ z^k \}_{\|k\|=0}^{\infty}$:

$$f(z, w) = \sum_{\|k\|=0}^{\infty} g_k(w) z^k .$$

This series converges absolutely in H. It is easily checked that $g_k(w) \in V(G)$ for all $\|k\| = 0, 1, 2, \ldots$.

Using the compactness principle for functions of several complex variables and the regularity (or equicontinuity following the terminology of Pietsch [5]) of an absolute basis in a Fréchet space we get that the above mentioned series converges absolutely in $(HG)_{n,N}$.

On the other hand, $g_k(w)$ can be devèlopped in a series in $V(G)$ with respect to the basis $\{ x_{m_1}^{(1)}(w_1) \ldots x_{m_N}^{(N)}(w_N) \}$.

Consequently,

$$f(z,w) = \sum_{\|k\|=0}^{\infty} \sum_{\|m\|0}^{\infty} \zeta_{km} z^k x_{m_1}^{(1)} (w_1)\dots x_{m_N}^{(N)} (w_N).$$

Using the regularity of the basis $\mathcal{X}$ we then establish that the latter double series converges absolutely in $(HG)_{n,N}$.

The uniqueness of the expansion can be verified easily Q. E. D.

Establishing an isomorphism between the completion of the tensor product of the spaces V_α ($\alpha = 1, \dots, n$), $V(G_\beta)$ ($\beta = 1, \dots, N$) and the space $(HG)_{n,N}$ yields the following generalization of Theorem 1.

THEOREM 2. *Let H be the space in Theorem 1, and let G_β be a domain in the w_β-plane such that there exists a basis in $V(G_\beta)$, $1 \leqslant \beta \leqslant N$. Further, let* $\Phi_\alpha \overset{df}{=} \left\{ \varphi_k^{(\alpha)} (z_\alpha) \right\}_{k=1}^{\infty} \subset V_\alpha$, $1 \leqslant \alpha \leqslant n$, $\psi_\beta \overset{df}{=} \left\{ \psi_m^{(\beta)} (w_\beta) \right\}_{m=1}^{\infty} \subset V(G_\beta)$, $1 \leqslant \beta \leqslant N$.

Then the system

$$\Phi \Psi \overset{df}{=} \left\{ \varphi_{k_1}^{(1)} (z_1)\dots \varphi_{k_1}^{(n)} (z_n) \psi_{m_1}^{(1)} (w_1)\dots \psi_{m_N}^{(N)} (w_N) \right\}_{k_1, \dots, k_n, m_1, \dots, m_N = 1}^{\infty}$$

is ARS in $(HG)_{n,N}$ if and only if each «one-dimensional» system is ARS in «its» space.

2.3. Theorem 2 from [3] on the absolute representation property for systems of analytic functions in polycylindrical domains can be generalized (in some sense) to the case of multi-dimensional domains as follows.

THEOREM 3. *Let W_j $(1 \leqslant j \leqslant n)$ be a domain in $\mathbf{C}^{m_j}$ such that there exists a basis in $V(W_j)$. Let $\Omega_j \overset{df}{=} \left\{ \omega_k^{(j)} (z_j) \right\}_{k=1}^{\infty} \subset V(W_j)$, where $z_j = \left(z_j^{(1)}, \dots, z_j^{(m_j)} \right)$, $j = 1, \dots, n$. Then the system $\prod_{j=1}^{n} \Omega_j$ is ARS in $V(W_1 \times \dots \times W_n)$ if and only if each component Ω_j is ARS in $V(W_j)$, $j = 1, \dots, n$.*

The proof is similar to that of Theorem 2 from [3].

Remark. As is known (see [6]), if W_j is a bounded convex domain in $\mathbf{C}^{m_j}$, then there exists a basis in $V(W_j)$. Therefore, Theorem 3 is valid for bounded convex domains W_j in $\mathbf{C}^{m_j}$ $(j = 1, \dots, n)$.

§3. PROPERTY OF INWARD CONTINUABILITY OF MULTI-DIMENSIONAL REPRESENTING SYSTEMS OF EXPONENTIALS

3.1. We recall some definitions from [7].

An RS (ARS) in $V(G)$, where G is a domain in $\mathbf{C}^n (n \geqslant 1)$, is said to be inward continuable from G to the subdomain $W \subset G$ if it is RS (ARS) in $V(W)$.

Without loss of generality, we may suppose $0 \in G$.

Further, a subdomain W of the convex domain G in $\mathbf{C}^n (n \geqslant 1)$ is said to be *convex-complementable* in G if there exists a convex compact set K_W in $\mathbf{C}^n$ such that $W + K_W = G$.

It is not hard to see that the domain W is convex-complementable in G if and only if the function $h(\zeta) = h_G(\zeta) - h_W(\zeta)$ is convex and finite. In particular, if G is a bounded convex domain then $dG\,(0 < d < 1)$ is convex-complementable in G.

Finally, it should also be mentioned that there exist different definitions for the notion of functions of completely regular growth (c.r.g.) of several variables. The « strong » definition belongs to Gruman (see, e.g., [8]), while the « weak » belongs to Azarin and Agranović (see, e.g., [9]. It is known that:

a) if $F(z)$ is a function of c.r.g. (in the « strong » sense), then the following « addition theorem of indicators » holds: Given any entire function of exponential type $S(z)$, the equality

$$\mathcal{L}^*_{FS}(z) = \mathcal{L}^*_F(z) + \mathcal{L}^*_S(z) \quad \forall z \in \mathbf{C}^n,$$

is satisfied, where $\mathcal{L}^*_f(\zeta)$ is a regularized radial indicator of the entire function $f(z)$ (see, e.g., [8]).

b) the « addition theorem of indicators » is equivalent to the « weak » definition but the two above mentioned definitions are not equivalent (see [9]).

3.2. Now let $\Lambda_j = \left\{\lambda_k^{(j)}\right\}_{k=1}^{\infty}$ $(1 \leqslant j \leqslant n)$ be finite nonzero complex numbers in the z_j — plane. Denote

$$\varepsilon_{\Lambda_j} = \left\{ e^{\lambda_k^{(j)} z_j} \right\}_{k=1}^{\infty} \quad (1 \leqslant j \leqslant n), \ \varepsilon_\Lambda = \prod_{j=1}^{n} \varepsilon_{\Lambda_j}$$

THEOREM 4. *If G is a finite convex domain in $\mathbf{C}^n$, and W is as ubdomain convex-complementable in G, then ε_Λ is inward continuable from G to W.*

Proof. There exists a convex compact set K_W in $\mathbf{C}^n$ such that $W + K_W = G$. For the sake of simplicity let us write K for K_W

According to a result of Sigurdsson (see [10]) There is a function of c. r. g. $a(\lambda)$ (in the « weak » sense) such that $\mathcal{L}^*_a(\zeta) = h_k(\zeta)$, $\zeta \in \mathbf{C}^n$. Moreover, by the above mentioned remarks the « addition theorem of indicators » hold.

We consider a functional $T \in V'(K)$ corresponding to $a(\lambda)$ and introduce a convolution operator $\check{T}$ by the formula

$$(\check{T}y)(x) = \langle y(x+\xi), T_\xi \rangle, \; \forall\, y \in V(W+K),$$

where for each fixed $x \in W$, the functional T acts at $y(x+\xi)$ as an element of the space $V(K)$.

As shown in [8], the operator $\check{T}$ is an epimorphism of $V(W+K)$ onto $V(W)$. Therefore, for any function $f(z) \in V(W)$, there is a function $y \in V(G)$ such that $(\check{T}y)(z) = f(z)$ for all $z \in W$.

Let ε_Λ be a RS in $V(G)$. In that case,

$$y(z) = \sum_{k_1,\dots,k_n=1}^{\infty} d_{k_1 \dots k_n} e^{\langle \tilde{\lambda}_k , z \rangle}, \quad \tilde{\lambda}_k = \left(\lambda^{(1)}_{k_1}, \dots, \lambda^{(n)}_{k_n}\right).$$

Furthermore, the series on the right hand side converges to $y(z)$ in $V(G)$. By the continuity of the operator $\check{T}$, the series $\sum_{k_1,\dots,k_n=1}^{\infty} d_{k_1 \dots k_n} \check{T}\left(e^{\langle \tilde{\lambda}_k, z \rangle}\right)$ converges in $V(W)$ to $(\check{T}y)(z) = f(z)$.

On the other hand,

$$\check{T}\left(e^{\langle \tilde{\lambda}_k, z \rangle}\right) = \left\langle e^{\langle \tilde{\lambda}_k, z+\xi \rangle}, T_\xi \right\rangle =$$

$$= e^{\langle \lambda_k, z \rangle} \left\langle e^{\langle \tilde{\lambda}_k, \xi \rangle}, T_\xi \right\rangle = e^{\langle \tilde{\lambda}_k, z \rangle} \cdot a(\tilde{\lambda}_k).$$

So, ε_Λ is RS in $V(W)$.

Repeating the same arguments and noting that the continuous operator $\check{T}$ maps any absolutely convergent series in the topology of $V(G)$ into an absolutety convergent series in the topology of $V(W)$, we see that if ε_Λ is ARS in $V(G)$, then ε_Λ is ARS in $V(W)$. Q.E.D.

REMARKS. 1) Theorem 4 was proved first for a less general case in our paper [11].

2. The proof of Theorem 4 is based on the onedimensional scheme used in [7].

3. In the case when G is a bounded convex domain in $\mathbf{C}^n$, while W is a convex-complementable subdomain in G, we have $\overline{G} = \overline{W} + K_W$. As shown in [8], the operator $\check{T}$ is an epimorphism of $V(\overline{G})$ onto $V(\overline{W})$.
Using the same method as that used for the proof of Theorem 4 we obtain the following result.

THEOREM 5. *If G is a bounded convex domain in $\mathbf{C}^n$, and W is a convex-complementable subdomain in G, then the system ε_Λ is inward continuable from $\overline{G}$ to $\overline{W}$.*

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