ACTA MATHEMATICA VIETNAMICA

Vol 11 N° 2 (1986)

TRANSLATIONS OF RELATIONAL SCHEMAS

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INTRODUCTION

In this paper we shall be concerned with a class of translations of relational schemas.

Starting from a given relational schema, translations make it possible to obtain simpler schemas, i.e. those with a less number of attributes and with shorter functional dependencies so that the key-finding problem becomes less cumbersome, etc.

On the other hand, from the set of keys of the run relational schema obtained in this way the corresponding keys of the original schema can be found by a single «translation».

In section 1 we introduce the notion of z-translation of a relational schema, give a classification of the relational schemas and investigate the characteristic properties of some classes of z-transformations.

In section 2 we study some properties of the so called nontranslatable relational schemas.

The notation used here is the same as in [1]; $\subset$ means strict inclusion.

SECTION 1

DEFINITION 1.1. *Let $S = \langle \Omega, F \rangle$ be a relational schema, where*

$\Omega = \{A_1, A_2, \dots, A_n\}$ *is the set of attributes,*

$F = \{L_i \to R_i \mid i = 1, 2, \dots, k;\ L_i,\ R_i \subseteq \Omega\}$

is the set of functional dependencies, and $Z \subseteq \Omega$, be an arbitrary subset of Ω. We define a new relational schema $\langle \Omega_1, F_1 \rangle$ by:

$$\Omega_1 = \Omega \setminus Z$$

$$F_1 = \{L_i \setminus z \to R_i \setminus Z \mid (L_i \to R_i) \in F,\ i = ç, \dots, k\}$$

Then $\langle \Omega_1, F_1 \rangle$ *is said to be obtained from* $\langle \Omega, F \rangle$ *by a Z-translation, and the notation*

$$\langle \Omega_1 F_1 \rangle = \langle \Omega, F \rangle - Z$$

is used.

Remarks

1. Depending on the characteristic properties of the class chosen, the corresponding class of translations has its own characteristic features.

2. With the Z-translation just defined above, a functional dependency of type $\emptyset \rightarrow Y$ may occur in $\langle \Omega_1, F_1 \rangle$ that has no ordinary semantic but carries information from the old relational schema to the new one.

In particular, the possibility that $\emptyset$ turns out to be a key of $\langle \Omega_1, F_1 \rangle$ is not excluded.

The next lemma is fundamental for the paper.

LEMMA 1. 1: *Let* $\langle \Omega, F \rangle$ *be a relational schema and*

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F\rangle - Z\rangle,\ Z \subseteq \Omega$$

then

a) $X \underset{F}{\rightarrow} Y$ *implies* $X \setminus Z \underset{F_1}{\rightarrow} Y \setminus Z$

b) $X \underset{F_1}{\rightarrow} Y$ *implies* $X \cup Z \underset{F}{\rightarrow} Y \cup Z$

where $X \underset{F}{\rightarrow} Y$ *means* $(X \rightarrow Y) \in F+$ *and similarly,* $X \underset{F_1}{\rightarrow} Y$ for $(X \rightarrow Y) \in F_1^+$.

Proof. For the part a) of the lemma, we shall prove that

$$X_F^+ \setminus Z \subseteq (X \setminus Z)_{F_1}^+. \qquad (1)$$

By the algorithm for finding the closure $X+$ of X in [2] with $X_F^{(0)} = X$, $(X \setminus Z)_F^{(0)} = X \setminus Z$ we have

$$X_F^{(0)} \setminus Z \subseteq (X \setminus Z)_{F_1}^{(0)},$$

Supposing that (1) holds for i, that is

$$X_F^{(i)} \setminus Z \subseteq (X \setminus Z)_{F_1}^{(i)}, \qquad (2)$$

we prove that (1) holds for $(i+1)$ as well.

Indeed we have

$$X_F^{(i+1)} \setminus Z = (X_F^{(i)} \cup (\bigcup_{\substack{L_J \subseteq X_F^{(i)} \\ (L_J \rightarrow R_J) \in F}} R_J)) \setminus Z = (X_F^{(i)} \setminus Z) \cup (\bigcup_{\substack{L_J \subseteq X_F^{(i)} \\ (L_J \rightarrow R_J) \in F}} R_J \setminus Z) \subseteq$$

$$\subseteq (X \setminus Z)^{(i)}_{F_i} \cup \Big(\bigcup_{L_J \subseteq X^{(i)}_F} (R_J \setminus Z)\Big)$$

(by virtue of the inductive assumption (2)).

On the other hand, from $L_J \subseteq X^{(i)}_F$ and the inductive assumption (2) we have:

$$L_J \setminus Z \subseteq X^{(i)}_F \setminus Z \subseteq (X \setminus Z)^{(i)}_{F_1}$$

Consequently:

$$X^{(i+1)}_F \setminus Z \subseteq (X \setminus Z)^{(i)}_{F_1} \cup \Big(\bigcup_{L_J \subseteq X^{(i)}_F} (R_J \setminus Z)\Big) \subseteq (X \setminus F)^{(i+1)}_{F_1} .$$

Thus (1) has been proved.

Now, it is well known that

$$X \xrightarrow[F]{} Y \Leftrightarrow Y \subseteq X^+_F$$

Hence, from $X \xrightarrow[F]{} Y$, we have:

$$Y \setminus Z \subseteq X^+_F \setminus Z \subseteq (X \setminus Z)^+_{F_1}$$

That is,

$$X \setminus Z \xrightarrow[F_1]{} Y \setminus Z$$

Similarly, for the part b) of the lemma, we shall prove by induction that

$$X^+_{F_1} \cup Z \subseteq (X \cup Z)^+_F . \qquad (3)$$

By the algorithm for finding the closure X^+ of X we have

$$X^{(0)}_{F_1} \cup Z \subseteq (X \cup Z)^{(0)}_F .$$

Supposing that (3) holds with (i), that is

$$X^{(i)}_{F_1} \cup Z \subseteq (X \cup Z)^{(i)}_F , \qquad (4)$$

we shall prove that (3) also holds for (i + 1).

Indeed we have: $X^{(i+1)}_{F_1} \cup Z = X^{(i)}_{F_1} \cup \Big(\bigcup_{L_J \setminus Z \subseteq X^{(i)}_{F_1}} (R_J \setminus Z)\Big) \cup Z =$

$$= (X^{(i)}_{F_1} \cup Z) \cup \Big(\bigcup_{L_J \setminus Z \subseteq X^{(i)}_{F_1}} (R_J \setminus Z)\Big) \subseteq (X \cup Z)^{(i)}_F \cup \Big(\bigcup_{L_J \setminus Z \subseteq X^{(i)}_{F_1}} R_J\Big)$$

(by the inductive assumption (4)).

On the other hand, from $L_J \setminus Z \subseteq X^{(i)}_{F_1}$ and (4) we have

$$L_J \subseteq X^{(i)}_{F_1} \cup Z \subseteq (X \cup Z)^{(i)}_F$$

Consequently:

$$X^{(i+1)}_{F_1} \cup Z \subseteq (X \cup Z)^{(i)}_F \cup \bigcup_{L_J / Z \subseteq X^{(i)}_{F_1}} (R_J) \subseteq (X \cup Z)^{(i+1)}_F .$$

Thus (3) has been proved.

From $X \underset{F_1}{\rightarrow} Y$ we have $Y \subseteq X^+_{F_1}$ hence

$$Y \cup Z \subseteq X^+_{F_2} \cup Z \subseteq (X \cup Z)^+_F$$

showing that: $X \cup Z \underset{F}{\rightarrow} Y \cup Z.$

The proof is complete.

DEFINITION 1.2 *Let $S = \langle \Omega, F \rangle$ be a relational schema. Let $\mathcal{K}(\Omega, F)$ be the set of all keys of S and*

$$H = \bigcup_{X_i \in \mathcal{K}(\Omega,F)} X_i, \qquad G = \bigcap_{X_i \in \mathcal{K}(\Omega,F)} X_i .$$

Now, we give a classification of the relational schemas as follows:

$$\mathcal{L}_0 = \{\langle \Omega, F \rangle \mid \langle \Omega, F \rangle \text{ is a relational schema}\},$$
$$\mathcal{L}_1 = \{\langle \Omega, F \rangle \in \mathcal{L}_0 \mid \Omega = L \cup R\},$$
$$\mathcal{L}_2 = \{\langle \Omega, F \rangle \in \mathcal{L}_0 \mid L \subseteq R = \Omega\},$$
$$\mathcal{L}_3 = \{\langle \Omega, F \rangle \in \mathcal{L}_0 \mid R \subseteq L = \Omega\},$$
$$\mathcal{L}_4 = \{\langle \Omega, F \rangle \in \mathcal{L}_0 \mid L = R = \Omega\}.$$

From the above classification, it is easily seen that:

$$\alpha)\ \mathcal{L}_4 \subseteq \mathcal{L}_3 \subseteq \mathcal{L}_1 \subseteq \mathcal{L}_0,$$
$$\beta)\ \mathcal{L}_4 \subseteq \mathcal{L}_2 \subseteq \mathcal{L}_1 \subseteq \mathcal{L}_0,$$
$$\gamma)\ \mathcal{L}_4 = \mathcal{L}_2 \cap \mathcal{L}_3.$$

We are now in a position to prove the following theorems.

THEOREM 1.1. *Let $\langle \Omega, F \rangle$ be a relational schema, $Z \subseteq G$; $\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - Z$. Then X is a key of $\langle \Omega_1, F_1 \rangle$ iff $X \cap Z = \varnothing$ and $X \cup Z$ is a key of $\langle \Omega, F \rangle$.*

Proof. We first prove the necessity. Suppose that X is a key of $\langle \Omega_1, F_1 \rangle$. Obviously $X \subseteq \Omega_1$, therefore $X \cap Z_1 = \varnothing$. Since X is a key of $\langle \Omega_1 F_1 \rangle$, $X \underset{F_1}{\rightarrow} \Omega_1$. Taking lemma 1.1 into account we get

$$X \cup Z \underset{F}{\rightarrow} \Omega_1 \cup Z = \Omega$$

showing that $X \cup Z$ is a superkey of $\langle \Omega, F \rangle$. Were $X \cup Z$ not a key of $\langle \Omega, F \rangle$ then there would exist a key $\overline{X}$ of $\langle \Omega, F \rangle$ such that

$$Z \subseteq \overline{X} \subset X \cup Z.$$

Consequently, there would exist an $X_1 \subset X$ such that

$$\bar{X} = X_1 \cup Z,\ X_1 \cap Z = \varnothing.$$

Since $\bar{X}$ is supposed to be a key of $\langle \Omega, F \rangle$, $X_1 \cup Z \underset{F}{\rightarrow} \Omega$.

Applying lemma 1. 1, clearly

$$(X_1 \cup Z) \setminus Z \underset{F_1}{\rightarrow} \Omega \setminus Z,$$

that is

$$X_1 \underset{F_1}{\rightarrow} \Omega_1,$$

This contradicts the hypothesis that $\bar{X}$ is a key of $\langle \Omega_1, F_1 \rangle$. Thus $X \cup Z$ is a key of $\langle \Omega, F \rangle$.

We now turn to the proof of sufficiency. Suppose that $X \cap Z = \phi$ and $X \cup Z$ is a key of $\langle \Omega, F \rangle$. We have to show that X is a key of $\langle \Omega_1, F_1 \rangle$.

Since $X \cup Z$ is a key of $\langle \Omega, F \rangle$ we have

$$X \cup Z \underset{F}{\rightarrow} \Omega.$$

By virtue of lemma 1. 1, we get

$$(X \cup Z) \setminus Z \underset{F_1}{\rightarrow} \Omega \setminus Z.$$

Consequently (from $X \cap Z = \phi$):

$$X \underset{F_1}{\rightarrow} \Omega_1,$$

showing that X is a superkey of $\langle \Omega_1, F_1 \rangle$. Assume that X is not a key of $\langle \Omega_1, F_1 \rangle$. Then, there would exist a key $\bar{X}$ of $\langle \Omega_1, F_1 \rangle$ such that

$$\bar{X} \subset X \text{ and } \bar{X} \underset{F_1}{\rightarrow} \Omega_1.$$

Applying lemma 1. 1, it follows:

$$\bar{X} \cup Z \underset{F}{\rightarrow} \Omega_1 \cup Z = \Omega,$$

where $\bar{X} \cup Z \subset X \cup Z$.

This contradicts the fact that $X \cup Z$ is a key of $\langle \Omega, F \rangle$.

Hence X is a key of $\langle \Omega_1, F_1 \rangle$. The proof is complete.

THEOREM 1.2. *Let* $\langle \Omega, F \rangle$ *is a relational schema,* $Z \subsetneqq \Omega$, $Z \cap H = \phi$ *and* $\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - Z$. *Then* X *is a key of* $\langle \Omega_1, F_1 \rangle$ *iff* X *is a key of* $\langle \Omega, F \rangle$.

Proof.

(i) (The necessity) Suppose that X is a key of $\langle \Omega_1, F_1 \rangle$. Obviously $X \underset{F_1}{\rightarrow} \Omega_1$.

By virtue of Lemma 1. 1, we have

$$X \cup Z \underset{F}{\rightarrow} \Omega_1 \cup Z = \Omega,$$

showing that $X \cup Z$ is a superkey of $\langle \Omega, F \rangle$. Hence, there exists a key $\overline{X}$ of $\langle \Omega, F \rangle$ such that $\overline{X} \subseteq X \cup Z$. Since $Z \cap H = \phi$ then $\overline{X} \cap Z = \phi$. From this, it is easy to see that $\overline{X} \subseteq X$. There are two possible cases:

a) $\overline{X} = X$. Then obviously X is a key of $\langle \Omega, F \rangle$.

b) $\overline{X} \subset X$. Since $\overline{X}$ is a key of $\langle \Omega, F \rangle$, $\overline{X} \underset{F}{\rightarrow} \Omega$.

Applying lemma 1. 1, we have

$$\overline{X} \setminus Z \underset{F_1}{\rightarrow} \Omega \setminus Z,$$

that is

$$X \underset{F_1}{\rightarrow} \Omega_1.$$

This contradicts the fact that X is a key of $\langle \Omega_1, F_1 \rangle$.

(ii) (The sufficiency). Suppose that X is a key of $\langle \Omega, F \rangle$. We have to prove that X is also a key of $\langle \Omega_1, F_1 \rangle$. We have, by the definition of keys,

$$X \underset{F}{\rightarrow} \Omega .$$

Applying Lemma 1.1:

$$X \setminus Z \underset{F_1}{\longrightarrow} \Omega \setminus Z = \Omega_1.$$

Since $Z \cap H = \varnothing$, it follows $X \cap Z = \varnothing$. Consequently,

$$X \underset{F_1}{\longrightarrow} \Omega_1$$

showing that X is a superkey of $\langle \Omega_1. F_1 \rangle$.

Now, assume the contrary that X is not a key of $\langle \Omega_1, F_1 \rangle$. Then there would exist a key $\overline{X}$ of $\langle \Omega_1, F_1 \rangle$ such that $\overline{X} \subset X$. Obviously

$$\overline{X} \underset{F_1}{\longrightarrow} \Omega_1.$$

We invoke lemma 1.1 to deduce

$$\overline{X} \cup Z \underset{F}{\rightarrow} \Omega_1 \cup Z = \Omega,$$

showing that $\overline{X} \cup Z$ is a superkey of $\langle \Omega, F \rangle$. Consequently, there exists a key $\overline{\overline{X}}$ of $\langle \Omega, F \rangle$ such that

$$\overline{\overline{X}} \subseteq \overline{X} \cup Z, \qquad \overline{\overline{X}} \cap Z = \varnothing.$$

From this $\quad \overline{\overline{X}} \subseteq \overline{X} \subset X.$

This contradicts the hypothesis that X is a key of $\langle \Omega, F \rangle$. The proof is complete.

Based on Theorems 1.1 and 1.2, in the following we investigate only the class of Z-translations with $Z \neq \varnothing$, $Z = Z_1 \cup Z_2$, $Z_1 \cap Z_2 = \varnothing$. $Z_1 \subset G$, $Z_2 \cap H = \varnothing$.

Bearing this in mind, if

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - Z$$

then applying Theorem 1.2 and 1.1 one after another to the Z_2-translation and the Z_1-translation, we have: X is a key of $\langle \Omega_1, F_1 \rangle$ if and only if $X \cap \varnothing = Z$ and $X \cup Z_1$ is a key of $\langle \Omega, F \rangle$. For the sake of convenience, we use in the sequel the notation

$$\langle \Omega, F \rangle \xrightarrow[p=(Z_1, Z_1)]{} \langle \Omega_1, F_1 \rangle,$$

where the meaning of p is obvious.

To continue, let us recall a result in [1]. Let $S = \langle \Omega, F \rangle$ be a relational schema, where

$$\Omega = \{ A_1, \dots, A_n \} \text{ — the set of attributes,}$$

$F = \{ L_i \to R_i \mid L_i, R_i \subset \Omega, i = 1, \dots, k \}$ — the set of functional dependencies.

Let us denote

$$L = \bigcup_{i=1}^{k} L_i, \quad R = \bigcup_{i=1}^{k} R_i.$$

Then, the necessary condition for which X is a key of S is that

$$\Omega \setminus R \subseteq X \subseteq (\Omega \setminus R) \cup (L \cap R).$$

For $V \subseteq \Omega$ we denote $\overline{V} = \Omega \setminus V$. It is easily seen that

$$\overline{L \cup R} \subseteq \Omega \setminus R \subseteq G$$
$$L \setminus R \subseteq \Omega \setminus R \subseteq G$$

$R \setminus L \subseteq \overline{H}$, consequently $(R \setminus L) \cap H = \varnothing$, and we have the following lemma:

LEMMA 1.2. *Let $S = < \Omega, F >$ be a relational schema, $Z \subsetneq G$, where G is the intersection of all the keys of S.*

Then $(Z^+ \setminus Z) \cap H = \varnothing$, where H is the union of all the keys of S.

Proof. Assume the contrary that

$$(Z^+ \setminus Z) \cap H \neq \varnothing.$$

Then, there would exist an attribute $A \in Z^+$, $A \overline{\in} Z$ and $A \in H$. Consequently, there exists a key X of $S = < \Omega, F >$ such that $A \in X$. Since $A \in Z^+$ and $A \overline{\in} Z$ we infer that $Z \subsetneq X \setminus A$. Hence

$$X \setminus A \xrightarrow{*} Z \xrightarrow{*} Z^+ \xrightarrow{*} A$$

with $A \in X$. This contradicts to the fact that X is a key of S. The proof is complete.

From the results mentioned just above the following theorems are obvious.

THEOREM 1.3. *Let $S = < \Omega, F >$ be a relational schema belonging to $\mathcal{L}_0$,*

$$< \Omega_1, F_1 > = < \Omega, F > - \overline{L \cup R}.$$

Then

$$< \Omega, F > \xrightarrow[\rho = (\overline{L \cup R}, \overline{L \cup R}]{} < \Omega_1, F_1 >$$

with

$$< \Omega_1, F_1 > \in \mathcal{L}_1.$$

Proof. As remarked above $\overline{L \cup R} \subseteq G$. Applying Theorem 1.1 to the Z-translation with $Z = \overline{L \cup R}$ we have

$$<\Omega, F> \xrightarrow[\rho = (\overline{L \cup R}, \overline{L \cup R})]{} < \Omega_1, F_1>.$$

Example 1. Let there be given $S = < \Omega, F >$ with $\Omega = \{a, b, c, d, e\}$, $F = \{ c \to d, d \to e \}$. We have $\overline{L \cup R} = ab$. Consider $<\Omega_1, F_1> = <\Omega, F> - ab$. Obviously $\Omega_1 = \{c, d, e\}$, $F_1 = \{c \to d, d \to e\}$.

It is easily seen that c is the unique key of $<\Omega_1, F_1>$ hence abc is the unique key of $<\Omega, F>$.

THEOREM 1.4. *Let $<\Omega, F>$ be a relational schema of $\mathcal{L}_0$.*

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - (\overline{L \cup R}) \cup (L \setminus R)).$$

Then

$$\langle \Omega, F \rangle \xrightarrow[\rho = (\overline{L \cup R} \cup (L \setminus R), \overline{L \cup R} \cup (L \setminus R))]{} \langle \Omega_1, F_1 \rangle$$

with

$$\langle \Omega_1, F_1 \rangle \in \mathcal{L}_2.$$

Proof. It is clear that

$$Z = \overline{L \cup R} \cup (L \setminus R) = \Omega \setminus R \subseteq G.$$

The theorem 1.4 now follows from applying Theorem 1.1 to the Z-translation.

THEOREM 1.5. *Let $S = \langle \Omega, F \rangle$ be a relational schema of $\mathcal{L}_0$,*

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - (\overline{L \cup R} \cup (R \setminus L)).$$

Th n

$$\langle \Omega, F \rangle \xrightarrow[\rho = (\overline{L \cup R} \cup (R \setminus L), \overline{L \cup R})]{} \langle \Omega_1, F_1 \rangle$$

with

$$\langle \Omega_1, F_1 \rangle \in \mathcal{L}_3.$$

Proof. As remarked above, $R \setminus L \subseteq \overline{H}$. Let $Z = \overline{L \cup R} \cup (R \setminus L) = Z_1 \cup Z_2$, where $Z_1 = \overline{L \cup R} \subseteq G$, $Z_2 = R \setminus L$, $Z_2 \cap H = \varnothing$.

The theorem 1.5 now follows from sequential applications of Theorems 1.2 and 1.1 one after another to the Z_2-translation and the Z_1-translation.

THEOREM 1.6. *Let $S = \langle \Omega, F \rangle$ be a relational schema of $\mathcal{L}_0$,*

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - (\overline{L \cup R} \cup (L \setminus R) \cup (R \setminus L)).$$

Then

$$\langle \Omega, F \rangle \xrightarrow[= (L \cup R \cup (L/R) \cup (R/L), \overline{L \cup R} \cup (L/R))]{} \langle \Omega_1, F_1 \rangle$$

with

$$\langle \Omega_1, F_1 \rangle \in \mathcal{L}_4.$$

Proof. Let $Z = \overline{L \cup R} \cup (L \setminus R) \cup (R \setminus L) = Z_1 \cup Z_2$. where

$$Z_1 = \overline{L \cup R} \cup (L \setminus R) = \Omega \setminus R \subseteq G,$$

$$Z_2 = R \setminus L \subseteq \overline{H} \text{ or equivalently } Z_2 \cap H = \varnothing.$$

It is obvious that $\langle \Omega_1, F_1 \rangle$ is obtained from $\langle \Omega, F \rangle$ by the Z-translation. The proof of Theorem 1.6 is straight-forward.

Similarly, we can prove the following theorems :

THEOREM 1.7. *Let $S = < \Omega, F >$ be a relational schema of $\mathcal{L}_1$,*

$$< \Omega_1, F_1 > = < \Omega, F > - (L \setminus R).$$

Then

$$< \Omega, F > \xrightarrow[= (L \setminus R, L \setminus R)]{} < \Omega_1, F_1 >,$$

where $< \Omega_1, F_1 > \in \mathcal{L}_2$.

THEOREM 1.8. *Let $S = < \Omega, F >$ be a relational schema of $\mathcal{L}_1$,*

$$< \Omega_1, F_1 > = < \Omega, F > - (R \setminus L).$$

Then

$$< \Omega, F > \xrightarrow[\rho = (R \setminus L, \varnothing)]{} < \Omega_1, F_1 >,$$

where

$$< \Omega_1, F_1 > \in \mathcal{L}_3.$$

THEOREM 1.9. *Let $S = < \Omega, F >$ be a relational schema of $\mathcal{L}_1$,*

$$< \Omega_1, F_1 > = < \Omega, F > - ((L \setminus R) \cup (R \setminus L)).$$

Then

$$< \Omega, F > \xrightarrow[\rho = ((L \setminus R) \cup (R \setminus L), L \setminus R)]{} < \Omega_1, F_1 >,$$

where

$$< \Omega_1, F_1 > \in \mathcal{L}_4.$$

THEOREM 1.10. *Let $< \Omega, F >$ be a relational schema of $\mathcal{L}_2$,*

$$< \Omega_1, F_1 > = < \Omega, F > - (R \setminus L).$$

Then

$$< \Omega, F > \xrightarrow[\rho = (R \setminus L), \varnothing)]{} < \Omega_1, F_1 >,$$

where

$$< \Omega_1, F_1 > \in \mathcal{L}_4.$$

THEOREM 1.11. *Let* $< \Omega, F >$ *be a relational schema of* $\mathcal{L}_3$,

$$< \Omega_1, F_1 > = < \Omega, F > - (L \setminus R).$$

Then

$$< \Omega, F > \xrightarrow[\rho = (L \setminus R, L \setminus R)]{} < \Omega_1, F_1 >,$$

where

$$< \Omega_1, F_1 > \in \mathcal{L}_4.$$

Now, the following theorem follows from Theorems 1.1, 1.2 and Lemma 1. 3.

THEOREM 1.12. *Let* $\langle \Omega, F \rangle$ *be a relational of* $\mathcal{L}_6$,

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - \{\overline{L \cup R} \cup (L \setminus R)^+ \cup (R \setminus L)\}.$$

Then

$$\langle \Omega, F \rangle \xrightarrow[\rho = (\overline{L \cup R} \cup (L \setminus R)^+ \cup (R \setminus L), \overline{L \cup R} \cup (L \setminus R))]{} \langle \Omega_1, F_1 \rangle,$$

where

$$\langle \Omega_1, F_1 \rangle \in \mathcal{L}_4.$$

Proof. Put $Z = \overline{L \cup R} \cup (L \setminus R) \cup [(L \setminus R)^+ \setminus (L \setminus R)] \cup (R \setminus L) = Z_1 \cup Z_2$,

where $Z_1 = \overline{L \cup R} \cup (L \setminus R) = \Omega \setminus R \subseteq G$,

$$Z_2 = [(L \setminus R)^+ \setminus (L \setminus R)] \cup (R \setminus L).$$

Clearly $Z_2 \cap H = \varnothing$. Applying Theorem 1.2 to

$$\langle \Omega', F' \rangle = \langle \Omega, F \rangle - Z_2,$$

and then, Theorem 1.1 to

$$\langle \Omega_1, F_1 \rangle = \langle \Omega' F' \rangle - Z_1,$$

the proof of Theorem 1.12 is easy.

Example 2. Let $\Omega = a\, b\, h\, g\, q\, m\, n\, v\, w\, k\, l$,

$$F = \{ a \to b,\ b \to h,\ g \to q,\ kv \to w,\ w \to vl \},$$

we have

$$L = abgkvw;\ R = bhqwvl;\ R \setminus L = hql;$$

$$L \setminus R = kga;\ (L \setminus R)^+ = kgabhq;\ \overline{L \cup R} = mn;$$

$$(R \setminus L) \cup (L \setminus R)^+ \cup \overline{(L \cup R)} = mnkgabhql$$

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - mnkgabhql = \langle wv, \{ v \to w, w \to v \} \rangle.$$

It is easily seen that v and w are keys of $\langle \Omega_1, F_1 \rangle$. On the other hand

$$\overline{(L \cup R)} \cup (L \setminus R) = mnkga$$

Consequently $mnkgav$ and $mnkgaw$ are keys of $\langle \Omega, F \rangle$.

SECTION 2.

In this section we investigate some properties of the so-called nontranslatable relational schemas.

DEFINITION 2.1. *Let* $S = \langle \Omega, F \rangle$ *be a relational schema. S is called translatable if and only if there exist certain sets* $Z_1, Z_2 \subseteq \Omega$ *such that:*

(i) $Z_1 \neq \emptyset$

(ii) *X is a key of* $\langle \Omega_1, F_1 \rangle$, *iff* $X \cap Z_2 = \emptyset$ *and* $X \cup Z_2$ *is a key of* $\langle \Omega, F \rangle$, *where* $\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - Z_1$. *Otherwise S is called nontranslatable.*

THEOREM 2.1. *Let* $S = \langle \Omega, F \rangle$ *be a translatable relational schema with* Z_1, Z_2 *as defined above. Then*

$$H \setminus G = H_1 \setminus G_1,$$

where H and G (and similarly H_1 *and* G_1*) are defined in definition 1.2.*

Proof. Let

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - Z_1.$$

Since X is a key of $\langle \Omega_1, F_1 \rangle$ iff $X \cap Z_2 = \emptyset$ and $X \cup Z_2$ is a key of $\langle \Omega, F \rangle$, it follows that:

$$H = H_1 \cup Z_2, \quad Z_2 \cap H_1 = \emptyset,$$
$$G = G_1 \cup Z_2, \quad Z_2 \cap G_1 = \emptyset,$$

hence

$$H \setminus G = (H_1 \cup Z_2) \setminus (G_1 \cup Z_2) = ((H \cup Z_2) \setminus Z_2) \setminus G_1 = H_1 \setminus G_1$$

(because $Z_2 \cap H_1 = \emptyset$).

Combining Theorems 1.1, 1.2 with Theorem 2.1, the following theorem is obvious:

THEOREM 2.2. *Let* $S = \langle \Omega, F \rangle$ *be a relational schema.* $\langle \Omega, F \rangle$ *is non translatable iff* $H = \Omega$ *and* $G = \emptyset$.

THEOREM 2.3. *Let* $S = \langle \Omega, F \rangle$ *be a relational schema,*

$$\langle \Omega_1, F_1 \rangle = \langle \Omega, F \rangle - (G \setminus \overline{H}).$$

Then:

a) $\langle \Omega, F \rangle \xrightarrow[\rho=(G \cup \overline{H}, G)]{} \langle \Omega_1, F_1 \rangle$.

b) $\langle \Omega_1, F_2 \rangle$ *is non translatable.*

c) $\langle \Omega_1, F_1 \rangle \in \mathcal{L}_4$.

Proof. Let $Z = G \cup \overline{H} = Z_1 \cup Z_2$, where $Z_1 = G \subseteq G$, $Z_2 = \overline{H}$ (clearly $Z_2 \cap H = \varnothing$).

Hence part a) of the theorem is obvious. To prove b), we have only to show that

$$G_1 = \varnothing \text{ and } H_1 = \Omega_1.$$

From a) it is clear that X is a key of $\langle \Omega_1, F_1 \rangle$ iff $X \cap G = \varnothing$ and $X \cup G$ is a key of $\langle \Omega, F \rangle$.

Therefore,
$$G = G \cup G_1, \quad G \cap G_1 = \varnothing$$
$$H = G \cup H_1, \quad G \cap H_2 = \varnothing.$$

Hence
$$G_1 = G \setminus G = \varnothing \text{ and } H_1 = H \setminus G.$$

On the other hand we have
$$\Omega_1 = \Omega \setminus (G \cup \overline{H}) = (\Omega \setminus \overline{H}) \setminus G = H \setminus G = H_1.$$

To prove c) we have to show that
$$L^1 = R^1 = \Omega_1,$$

where L^1 and R^1 are the union of all the left sides and right sides of all functional dependencies of F_1, respectively.

It is known [1] that
$$\Omega_1 \setminus R^1 \subseteq G_1 = \varnothing.$$

On the other hand
$$R^1 \subseteq \Omega_1.$$

Hence
$$R^1 = \Omega_1.$$

There remained to prove $L^1 = \Omega_1$. Were this false, there would exist an $A \in \Omega_1 \setminus L^1$. Since $R^1 = \Omega_1$, we have $A \in R^1$ and $A \overline{\in} L^1$.

From $\Omega_1 = H_1$ there exists a key X of $\langle \Omega_1, F_1 \rangle$ such that
$$A \in X \text{ and } X \xrightarrow{*} \Omega_1.$$

Since $A \overline{\in} L^1$ it follows from [1] that
$$X \setminus A \xrightarrow{*} \Omega_1 \setminus A.$$

Evidently
$$L^1 \subseteq \Omega_1 \setminus A$$

and hence,
$$X \setminus A \xrightarrow{*} \Omega_1 \setminus A \xrightarrow{*} L \xrightarrow{*} R^1 \xrightarrow{*} A.$$

This contradicts the fact that X is a key of $\langle \Omega_1, F_2 \rangle$, hence $L^1 = \Omega_1$. The proof is complete.

From the proof of c) we conclude that all non translatable relational schemas are of type $\mathcal{L}_4$.

THEOREM 2.4. *Let $S = \langle \Omega, F \rangle$ be a relational schema from $\mathcal{L}_4$ satisfying the following conditions:*

(i) $L_i \cap R_i = \emptyset \quad \forall i = 1, 2, \ldots, k$,

(ii) *for each L_i, $i = 1, \ldots, k$ there exists a key X_i such that $L_i \subsetneq X_i$.*

Then $\langle \Omega, F \rangle$ is a nontranslatable relational schema.

Proof. We have to prove that $H = \Omega$ and $G = \emptyset$.
In fact, from $\langle \Omega, F \rangle \in \mathcal{L}_4$ we have $L = R = \Omega$. By virtue of the hypothesis of the theorem we have

$$\Omega = L = \bigcup_{i=1}^{k} L_i \subseteq \bigcup_{i=1}^{k} X_i \subseteq H \subsetneq \Omega$$

Consequently, $H = \Omega$.

To prove $G = \emptyset$ we first show that if $L_i \to R_i$ and X_i is a key such that $L_i \subsetneq X_i$ then $X_i \cap R_i = \emptyset$. Assume the contrary that $X_i \cap R_i \neq \emptyset$. Then, there would exist an $A \in X_i \cap R_i$. Since $L_i \cap R_i = \emptyset$ clearly $A \notin L_i$. Therefore $L_i \subseteq X_i \setminus A$.

On the other hand

$$X_i \setminus A \xrightarrow{*} L_i \xrightarrow{*} R_i \xrightarrow{*} A,$$

showing that X is not a key of $< \Omega, F >$. We thus arrive at a contradiction. From $X_i \cap R_i = \emptyset$, it follows:

$$X_i \subseteq \Omega \setminus R_i.$$

Thus

$$G \subseteq \bigcap_{i=1}^{k} X_i \subseteq \bigcap_{i=1}^{k} (\Omega \setminus R_i) = \Omega \setminus \bigcup_{i=1}^{k} R_i.$$

Since $R = \Omega$ clearly $G \subseteq \Omega \setminus \Omega = \emptyset$ showing that $G = \emptyset$. The proof is complete.

Acknowledgement. The authors would like to take this opportunity to express their sincere thanks to Prof. Dr. J. Demetrovics and Dr. A. Békéssy for their valuable comments and suggestions.

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Received January 28, 1985

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