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$\otimes$-STRICT ACU CATEGORIES

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0. INTRODUCTION

In [6] it is proved that every $\otimes$ - ACU category is $\otimes$ - ACU equivalent to a $\otimes$-ACU category, which is a $\otimes$-strict AU category. But we cannot prove that every $\otimes$ - ACU category is $\otimes$ - ACU equivalent to a $\otimes$ - strict ACU category, because for it the necessary condition is that the commutativity constraint satisfies the condition :

$$c_{A,A} = id_{A \otimes A}, \text{ for all } A \in Ob\underline{C}. \tag{0.1}$$

The purpose of this paper is to prove that every $\otimes$ – ACU category $\underline{C}$, in which the commutativity constraint c satifies the condition (0. 1), is $\otimes$ - ACU equivelent to a $\otimes$-strict ACU category. But by the result which is obtained in [6], it suffices to prove the above assertion for a $\otimes$-ACU category which is strict AU and the commutativity constraint satisfies the condition (0. 1).

1. $\otimes$ - STRICT ACU CATEGORIES

Let $\underline{C}$ be a $\otimes$ - ACU category, which is strict AU and the commutativity constraint satisfies the condition (0.1) i.e

$$c_{A,A} = id_{A \otimes A}, \text{ for all } A \in Ob\underline{C}.$$

We shall construct a $\otimes$-strict ACU category which is denoted by $\underline{M}\langle\underline{C}\rangle$.

First, we introduce some notations. An object of $\underline{C}$, which has the form $A_1 \otimes ... \otimes A_n$, $A_i \neq 1$ for $i = 1,..., n$, is said to be a *product*, each A_i of this product called factor. An object $A \neq 1$ is also said to be a product of one factor.

We denote the class of all objects $A \neq \underline{1}$ of $\underline{C}^*$ by C^* and consider the class $M\langle C^* \rangle$ of all functions

$$F : C^* \rightarrow N,$$

from C^* to the set N of all natural numbers, such that $F(A) = 0$ for all $A \in C^*$, except for a finite numbers. Thus each $F \in M\langle C^* \rangle$ defines a finite subse $\mathcal{A}$ of C^* and a family $(n_A)_{A \in \mathcal{A}}$ of natural numbers $n_A \neq 0$. Conversely, a pair ($\mathcal{A}$,

$(n_A)_{A \in \mathcal{A}}$, in which $\mathcal{A}$ is a finite subset of C^* and $(n_A)_{A \in \mathcal{A}}$ is a family of natural numbers, such that $n_A \neq 0$ for all $A \in \mathcal{A}$, defines a unique function $F \in M\langle C^* \rangle$.

Suppose that $F \in M\langle C^* \rangle$ defines a pair $(\mathcal{A}, (n_A)_{A \in \mathcal{A}})$.

In the case $\mathcal{A} \neq \phi$ we consider all products in $\underline{C}$, which contain n_A factors A, for all $A \in \mathcal{A}$. The number of these products is finite. We choose one of these products and call it *product of F*. For example, F is function defined as follows:

$F(A) = 2$, $F(B) = 1$, $F(C) = 0$ if $C \neq A$ and $C \neq B$. Consider all products which contain two factors A and one factor B. These are $A \otimes A \otimes B$, $A \otimes B \otimes A$, $B \otimes A \otimes A$. In the set of these products we choose, for instance, $A \otimes A \otimes B$ as the product of F.

If F is a function such that $F(A) = 1$ and $F(B) = 0$ for $B \neq A$, then the product of F is A.

In the case $\mathcal{A} = \phi$, we say $\underline{1}$ is the product of F.

We denote the product of F by $\otimes F$.

Thus, each $F \in M\langle C^* \rangle$ has a unique product.

When we write $(\otimes F) \otimes (\otimes G)$ we mean that this is the product, in which the first factor is the product of F and the rest of G. In general, it is not a product of some function in $M\langle C^* \rangle$.

Now we consider the triplets, which have the forms (F, G, u), where $u : \otimes F \to \otimes G$ is a morphism from the product of F to the product of G. We have the following

Proposition 1.1. We can define a category $M\langle C \rangle$ as follows:

$$\mathrm{Ob}M\langle C \rangle = M\langle C^* \rangle,$$

$$\mathrm{Hom}(F, G) = \{(F, G, u) \mid u : \otimes F \to \otimes G\}, \tag{1.1.1}$$

the composition of two morphisms is defined by the following relation:

$$(G, H, v)(F, G, u) = (F, H, vu) \tag{1.1.2}$$

and the identities have the forms $(F, F, id_{\otimes F})$ (1.1.3)

Proof. In fact, we have:

$$(G, H, w)((F, G, v)(E, F, u)) = (G, H, w)(E, G, vu) = (E, H, w(vu)) =$$
$$= (E, H, (wv)u) = (F, H, wv)(E, F, u) = ((G, H, w)(F, G, v))(E, F, u);$$
$$(F, G, u)(F, F, id_{\otimes F}) = (F, G, u.\, id_{\otimes F}) = (F, G, u);$$
$$(G, G, id_{\otimes F})(F, G, u) = (F, G, id_{\otimes F}.u) = (F, G, u);$$

Proposition 1.2. $\underline{M}\langle \underline{C} \rangle$ is a $\otimes$-category with the multiplication defined by the following relations:

$$(F \otimes G)(A) = F(A) + G(A), \ A \in C^* \tag{1.2.1}$$

$$(E, G, u) \otimes (F, H, v) = (E \otimes F, G \otimes H, y(u \otimes v)x^{-1}), \tag{1.2.2}$$

where $x : (\otimes E) \otimes (\otimes F) \to \otimes(E \otimes F)$

$y : (\otimes G) \otimes (\otimes H) \to \otimes(G \otimes H)$

are the morphisms built up from the morphisms c, id and $\otimes$ in $\underline{C}$.

Proof. First from (1.1.3) and (1.2.2) it follows:

$$\mathrm{id}_{\otimes F} \otimes \mathrm{id}_G = (F, F, \mathrm{id}_{\otimes F}) \otimes (G, G, \mathrm{id}_{\otimes G}) =$$

$$F \otimes G, F \otimes G, x(\mathrm{id}_{\otimes F} \quad \mathrm{id}_{\otimes G})x^{-1}) = (F \otimes G, F \otimes G, x(\mathrm{id}_{(\otimes F)\otimes(\otimes G)})x)$$

$$(F \otimes G, F \otimes G, xx^{-1})\ (F \otimes G, F \otimes G, \mathrm{id}_{\otimes(F \otimes G)}) = \mathrm{id}_{(F \otimes G)},$$

(where $x : (\otimes F) \otimes (\otimes G) \to \otimes(F \otimes G)$ is a morphism built up from the morphisms c, id, and $\otimes$ in $\underline{C}$).

Furthermore, let

$$\alpha = (E, G, u) : E \to G,\ \beta = (F, H, v) : F \to H,$$
$$\gamma = (G, K, w) : G \to K,\ \delta = (H, L, t) : H \to L$$

and

$$x : (\otimes E) \otimes (\otimes F) \to \otimes(E \otimes F),$$
$$y : (\otimes G) \otimes (\otimes H) \to \otimes(G \otimes H),$$
$$z : (\otimes K) \otimes (\otimes L) \to \otimes(K \otimes L).$$

are the morphisms built up from the morphisms c, id, and $\otimes$ in C, we have:

$$(\gamma \otimes \delta)(\alpha \otimes \beta) = ((G, K, w) \otimes (H, L, t))\,((E, G, u) \otimes (F, H, v)) =$$
$$= (G \otimes H, K \otimes L, z(w \otimes t)y^{-1})\,(E \otimes F, G \otimes H, y(u \otimes v)x^{-1}) =$$
$$= (E \otimes F, K \otimes L, z(w \otimes t)(u \otimes v)x^{-1}) =$$
$$= (E \otimes F, K \otimes L, z(wu \otimes tv)x^{-1}) =$$
$$= (E, K, wu) \otimes (F, L, tv) =$$
$$= ((G, K, w)\,(E, C, u)) \otimes ((H, L, t)\,(F, H, v)) = \gamma\alpha \otimes \delta\beta.$$

Proposition 1.3. In $\underline{M}\langle \underline{C} \rangle$

$F \otimes (G \otimes H) = (F \otimes G) \otimes H$, for all $F, G, H \in \mathrm{ob}\underline{M}\langle \underline{C} \rangle$

and $a_{F,G,H} = \mathrm{id}_{F \otimes G \otimes H} : F \otimes (G \otimes H) \to (F \otimes H) \otimes G$

is the associative constraint.

Proof. First, we have:

$$(F \otimes (G \otimes H))(A) = F(A) + (G \otimes H)(A) = F(A + (G(A) + H(A)) =$$
$$= (F(A) + G(A)) + H(A) = (F \otimes G)(A) + H(A) =$$
$$((F \otimes G) \otimes H)(A),\ A \in C^*.$$

Now we prove that $a_{F,G,H} = \mathrm{id}_{F \otimes G \otimes H}$ is an isomorphism of trifunctors. Let

$$\alpha = (F, F', u),\ \beta = (G, G', v),\ \gamma = (H, H', w)$$

and

$$t : (\otimes G) \otimes (\otimes H) \to \otimes(G \otimes H),$$
$$x : (\otimes G') \otimes (\otimes H') \to \otimes(G' \otimes H'),$$
$$y : (\otimes F) \otimes (\otimes(G \otimes H)) \to \otimes(F \otimes (G \otimes H))$$
$$z : (\otimes F') \otimes (\otimes(G' \otimes H')) \to \otimes(F' \otimes (G' \otimes H')),$$

are the morphisms built up from the morphisms c, id and $\otimes$ in $\underline{C}$, we have:

$$\alpha \otimes (\beta \otimes \gamma) = (F, F', u) \otimes ((G, G', v) \otimes (H, H', w)) =$$
$$= (F, F', u) \otimes (G \otimes H, G' \otimes H', x(v \otimes w)t^{-1}) =$$
$$= (F \otimes (G \otimes H), F' \otimes (G' \otimes H'), z(u \otimes x(v \otimes w)t^{-1})y^{-1}) =$$
$$= (F \otimes (G \otimes H), F' \otimes (G' \otimes H'), z(\mathrm{id} \otimes x)(u \otimes (v \otimes w))(\mathrm{id} \otimes t^{-1})y^{-1}) =$$
$$= ((F \otimes G) \otimes H),)F' \otimes G') \otimes H', z(\mathrm{id} \otimes x)((u \otimes v) \otimes w)(\mathrm{id} \otimes t^{-1})y^{-1})$$

since $u \otimes (v \otimes w) = (u \otimes v) \otimes w$ in the $\otimes$-strict AU category $\underline{C}$.

In the other hand, if;

$p : (\otimes F) \otimes (\otimes G) \to \otimes(F \otimes G)$,

$q : (\otimes F') \otimes (\otimes G' \to \otimes(F' \otimes G')$,

$r : (\otimes (F \otimes G)) \otimes (\otimes H) \to \otimes ((F \otimes G) \otimes H)$,

$s : (\otimes (F' \otimes G')) \otimes (\otimes H') \to \otimes ((F' \otimes G') \otimes H')$,

we have:

$$\begin{aligned}(\alpha \otimes \beta) \otimes \Upsilon &= ((F, F', u) \otimes (G, G', v)) \otimes (H, H', w) = \\ &= (F \otimes G, F' \otimes G', q(u \otimes v)p^{-1}) \otimes (H, H' w) = \\ &= ((F \otimes G) \otimes H, F' \otimes G') \otimes H', s(q(u \otimes v)p^{-1} \otimes w)r^{-1}) = \\ &= ((F \otimes G) \otimes H, (F' \otimes G') \otimes H', s(q \otimes id)((u \otimes v) \otimes w)(p \otimes id)r^{-1}).\end{aligned}$$

Since C is a $\otimes$-strict AU category,

$(\otimes F) \otimes ((\otimes G) \otimes (\otimes H)) = ((\otimes F) \otimes (\otimes G)) \otimes (\otimes H)$,

$(\otimes F') \otimes ((\otimes G') \otimes (\otimes H')) = ((\otimes F') \otimes (\otimes G')) \otimes (\otimes H')$,

and the morphisms $y(id \otimes t)$ and $r(p \otimes id)$ are the morphisms from $(\otimes F) \otimes ((\otimes G) \otimes (\otimes H))$ to $\otimes (F \otimes G \otimes H)$ built up from c, id and $\otimes$ in $\underline{C}$. Hence

$$y(id \otimes t) = r(p \otimes id).$$

Similarly, we have:

$$z(id \otimes x) = s(q \otimes id).$$

Thus

$$\alpha \otimes (\beta \otimes \Upsilon) = (\alpha \otimes \beta) \otimes \Upsilon.$$

Proposition 1.4. In $\underline{M} \langle \underline{C} \rangle$,

$$F \otimes G = G \otimes F$$

and $c_{F, G} = id_{F \otimes G} : F \otimes G \to G \otimes F$ is the commutativity constraint.

Proof. In fact,

$(F \otimes G)(A) = F(A) + G(A) = G(A) + F(A) = (G \otimes F)(A)$, $A \in C^*$.

Furthermore, let $\alpha = (F, F', u)$, $\beta = (G, G', v)$, and

$x : (\otimes F) \otimes (\otimes G) \to \otimes (F \otimes G)$, $y : (\otimes F') \otimes (\otimes G') \to \otimes (F' \otimes C')$

are the morphisms built up from c, id and $\otimes$ in $\underline{C}$, we have:

$$\begin{aligned}\alpha \otimes \beta &= (F, F', u) \otimes (G. G', v) = (F \otimes G, F' \otimes G', y(u \otimes v) x^{-1}) = \\ &= (F \otimes G, F' \otimes G', yc_{\otimes G', \otimes F'} (v \otimes u) c_{\otimes F, \otimes G}\, x^{-1}).\end{aligned}$$

But $yc_{\otimes G', \otimes F'}$ and $xc_{\otimes G, \otimes F}$ also are the morphisms built up from c, id and $\otimes$ in $\underline{C}$,

$$\alpha \otimes \beta = (F \otimes G, F' \otimes G', yc_{\otimes G', \otimes F'} (v \otimes u) c_{\otimes F, \otimes G}\, x^{-1}) = \beta \otimes \alpha.$$

i.e $id_{F \otimes G}$ is an isomorphism of bifunctors.

Proposition 1.5. In $\underline{M} \langle \underline{C} \rangle$

$$F \otimes \Gamma_1 = F = \Gamma_1 \otimes F,$$

where Γ_1 is a function such that $\Gamma_1(A) = 0$ fon all $A \in C^*$.

and $g_F = id_F = d_F$ are isomorphisms of functors, i.e $(\Gamma_{\underline{1}}, id, id)$ is an unity constraint.

Proof. We have:

$$(F \otimes \Gamma_{\underline{1}})(A) = F(A) + \Gamma_{\underline{1}}(A) = F(A) + 0 = F(A) = 0 + F(A) =$$
$$= \Gamma_{\underline{1}}(A) + F(A) = (\Gamma_{\underline{1}} \otimes F)(A), \; A \in C^*.$$

Now, let $\alpha = (F, F', u)$, we have:

$$\alpha \otimes id_{\Gamma_{\underline{1}}} = (F, F', u) \otimes (\Gamma_{\underline{1}}, \Gamma_{\underline{1}}, id_{\underline{1}}) = (F \otimes \Gamma_{\underline{1}}, F' \otimes \Gamma_{\underline{1}}, y(u \otimes id_{\underline{1}})\, x^{-1}),$$

where $x : \underline{1} \otimes (\otimes F) = (\otimes \Gamma_{\underline{1}}) \otimes (\otimes F) \to \otimes (\Gamma_{\underline{1}} \otimes F) = \otimes F,$

$$y : \underline{1} \otimes (\otimes F') = (\otimes \Gamma_{\underline{1}}) \otimes (\otimes F') \to \otimes (\Gamma_{\underline{1}} \otimes F') = \otimes F',$$

are the morphisms built up from the morphisms c, id and $\otimes$ in $\underline{C}$. Cince $\underline{C}$ is $\otimes$ - strict AU category, then:

$$u \otimes id_{\underline{1}} = u, \; \underline{1} \otimes (\otimes F) = \otimes F, \; \underline{1} \otimes (\otimes F') = \otimes F',$$
$$x = id_{\otimes F}, \; y = id_{\otimes F'}.$$

It follows:

$$\alpha \otimes id_{\Gamma_{\underline{1}}} = (F, F', u) = \alpha.$$

Finally, it is easy to see that $\underline{M}\langle\underline{C}\rangle$ is $\otimes$ – strict ACU category, i.e we have

Proposition 1.6. $\underline{M}\langle\underline{C}\rangle$ is a $\otimes$ - strict ACU category.

2. THEOREM

Now we prove the main theorem of this paper,

Theorem 2.1. Let $\underline{C}$ be a $\otimes$-ACU category which is strict AU and the commutativity constraint c satisfies the condition (0. 1). i . e

$$c_{A,A} = id_{A \otimes A}, \text{ foa all } A \in Ob\underline{C}.$$

Then $\underline{C}$ is $\otimes$ - ACU equivalent to the $\otimes$ - strict ACU category $\underline{M}\langle\underline{C}\rangle$.

Proof. We define the functor

$$\Gamma : \underline{C} \to \underline{M}\langle\underline{C}\rangle$$

by the following relations:

$$\Gamma(A) = \Gamma_A, \tag{2.1.1}$$

where Γ_A is the function such that $\Gamma_A(A) = 1$, $\Gamma_A(B) = 0$ foı all $B \neq A$, if $A \neq 1$; and Γ_1 is the function such that $\Gamma_1(A) = 0$ for all $A \in C^*$.

$$\Gamma(u) = (\Gamma_A, \Gamma_B, u) : u \;\; A \to B. \tag{2.1.2}$$

It is easy to see that Γ so defined isa functor. In fact,

$$\Gamma(id_A) = (\Gamma_A, \Gamma_A, id_A) = id_{\Gamma A}, \Gamma(vu) = (\Gamma_A, \Gamma_C, vu) = (\Gamma_B, \Gamma_C, v)(\Gamma_A, \Gamma_B, u) = \Gamma(v)\Gamma(u).$$

For a pair $A, B \in Ob\underline{C}$, we define an isomorphism of bifunctors $\breve{\Gamma}_{A,B}$ as follows;

$$\breve{\Gamma}_{A,B} = (\Gamma_{A\otimes B}, \Gamma_A \otimes \Gamma_B, x), \tag{2.1.3}$$

where'

$$x = \begin{cases} id_{A\otimes B}, & \text{if } A\otimes B = \otimes(\Gamma_A\otimes\Gamma_B), \\ c_{A,B}, & \text{if } B\otimes A = \otimes(\Gamma_A\otimes\Gamma_B). \end{cases}$$

It follows from (2. 1. 3):

$$\breve{\Gamma}_{A,1} = (\Gamma_{A\otimes 1}, \Gamma_A\otimes\Gamma_1, id_A) = (\Gamma_A, \Gamma_A, id_A) = \Gamma_{1\otimes A}, \Gamma_1\otimes L_A, id_A) = \breve{\Gamma}_{1,A} \tag{2.1.4}$$

beccause in $\underline{M}\langle\underline{C}\rangle$ $\Gamma_A\otimes\Gamma_1 = \Gamma_A = \Gamma_1\otimes\Gamma_A$ and in $\underline{C}$ $A\otimes\underline{1} = A = \underline{1}\otimes A$, $c_{1,A} = id_A = c_{A,1}$.

From (0. 1) and (2.1.3) it follows:

$$\breve{\Gamma}_{A,A} = (\Gamma_{A\otimes A}, \Gamma_A\otimes\Gamma_A, id_{A\otimes A}),$$

$\breve{\Gamma}$ is an isomorphism of bifunctors. In fact, first it is easy to see that the inverse of $\breve{\Gamma}_{A,B}$ is

$$\breve{\Gamma}^{-1}_{A,B} = (\Gamma_A\otimes\Gamma_B, \Gamma_{A\otimes B}, x^{-1}) \tag{2.1.5}$$

Moreover, suppose $u: A\to C$, $v: B\to D$ are the morphisms in $\underline{C}$. Applying the relations (1.1.2). (2.1.2) and (2.1.5) we obtain:

$$\begin{aligned} \Gamma(u)\otimes\Gamma(v) &= (\Gamma_A, \Gamma_C, u)\otimes(\Gamma_B, \Gamma_D, v) = (\Gamma_A\otimes\Gamma_B, \Gamma_C\otimes\Gamma_D, y(u\otimes v)x^{-1}) = \\ &= (\Gamma_{C\otimes D}, G_C\otimes\Gamma_D, y)(\Gamma_{A\otimes B}, \Gamma_{C\otimes D}, u\otimes v)(\Gamma_A\otimes\Gamma_B, \Gamma_{A\otimes B}, x^{-1}) \\ &= \breve{\Gamma}_{C,D}\,\Gamma(u\otimes v)\,\breve{\Gamma}^{-1}_{A,B}. \end{aligned}$$

Thus $(\Gamma, \breve{\Gamma})$ is a $\otimes$-functor. It is compatible with the associativity constraints. In fact, it follows from the relations (1.1.2), (1.1.3), (1.2.2), (2.1.2) and (2.1.5):

$$\begin{aligned} (\breve{\Gamma}_{A,B}\otimes id_{\Gamma C})\breve{\Gamma}_{A\otimes B,C} &= ((\Gamma_{A\otimes B}\,\Gamma_A\otimes\Gamma_B, x)\otimes(\Gamma_C, \Gamma_C, id_C))(\Gamma_{A\otimes B\otimes C}, \Gamma_{A\otimes B}\otimes\Gamma_C, y) = \\ &= (\Gamma_{A\otimes B}\otimes\Gamma_C, \Gamma_A\otimes\Gamma_B\otimes\Gamma_C, z(x\otimes id_C)y^{-1})(\Gamma_{A\otimes B\otimes C}, \Gamma_{A\otimes B}\otimes\Gamma_C, y) = \\ &= (\Gamma_{A\otimes B\otimes C}, \Gamma_A\otimes\Gamma_B\otimes\Gamma_C, z(x\otimes id_C)), \end{aligned}$$

where $x: A\otimes B\to\otimes(\Gamma_A\otimes\Gamma_B)$, $y: (A\otimes B)\otimes C\to\otimes(\Gamma_{A\otimes B}\otimes\Gamma_C)$,

$$z: (\otimes(\Gamma_A\otimes\Gamma_B))\otimes\Gamma_C \to \otimes(\Gamma_A\otimes\Gamma_B\otimes\Gamma_C)$$

are the morphisms built up from the morphisms c, id and $\otimes$ in $\underline{C}$.

On the other hand,

$$\begin{aligned} (id_{\Gamma A}\otimes\breve{\Gamma}_{B,C})\breve{\Gamma}_{A,B\otimes C} &= ((\Gamma_A, \Gamma_A, id_A)\otimes(\Gamma_{B\otimes C}, \Gamma_B\otimes\Gamma_C, t))\breve{\Gamma}_{A,B\otimes C} = \\ &= (\Gamma_A\otimes\Gamma_{B\otimes C}, \Gamma_A(\Gamma_B\otimes\Gamma_C), v(id_A\otimes t)u^{-1})(\Gamma_{A\otimes(B\otimes C)}, \Gamma_A\otimes\Gamma_{B\otimes C}, u) \\ &= (\Gamma_{A\otimes B\otimes C}, \Gamma_A\otimes\Gamma_B\otimes\Gamma_C, v(id_A\otimes t)), \end{aligned}$$

where $t: B\otimes C\to\otimes(\Gamma_B\otimes\Gamma_C)$, $u: A\otimes(B\otimes C)\to\otimes(\Gamma_A\otimes\Gamma_B\otimes\Gamma_C)$,

$$v: A\otimes(\otimes(\Gamma_B\otimes\Gamma_C))\to\otimes(\Gamma_A\otimes\Gamma_B\otimes\Gamma_C.$$

are the morphisms built up from the morphisms c, id and $\otimes$ in $\underline{C}$.

Since $z(x\otimes id_C)$ and $v(id_A\otimes t)$ are morphisms from $(A\otimes B)\otimes C$ to $\otimes(\Gamma_A\otimes\Gamma_B\otimes\Gamma_C)$ we have

$$z(x\otimes id_C) = v(id_A\otimes t).$$

Therefore

$(\Gamma_{A\otimes B\otimes C}, \Gamma_A\otimes\Gamma_B\otimes\Gamma_C, z(x\otimes id_C)) = (\Gamma_{A\otimes B\otimes C}, \Gamma_A\otimes\Gamma_B\otimes\Gamma_C, v(id_A\otimes t))$, i.e

$$(\breve{\Gamma}_{A,B}\otimes id_{\Gamma C})\breve{\Gamma}_{A\otimes B,C} = (id_{\Gamma A}\otimes\breve{\Gamma}_{B,C})\breve{\Gamma}_{A,B\otimes C}$$

or the following diagram is commutative:

$$\begin{array}{ccccc}
\Gamma(A\otimes(B\otimes C)) & \xrightarrow{\breve{\Gamma}_{A,B\otimes C}} & \Gamma A\otimes\Gamma(B\otimes C) & \xrightarrow{id_{\Gamma A}\otimes\breve{\Gamma}_{B,C}} & \Gamma A\otimes(\Gamma B\otimes\Gamma C) \\
\| & & & & \| \\
\Gamma((A\otimes B)\otimes C) & \xrightarrow{\breve{\Gamma}_{A\otimes B,C}} & \Gamma(A\otimes B)\otimes\Gamma C & \xrightarrow{\breve{\Gamma}_{A,B}\otimes id_{\Gamma C}} & (\Gamma A\otimes\Gamma B)\otimes\Gamma C
\end{array}$$

Now we prove that $(\Gamma, \breve{\Gamma})$ is compatible with the commutativity constraints. It follows from (2. 1. 2), (2. 1. 3)

$$\breve{\Gamma}_{B,A}\Gamma(c_{A,B}) = (\Gamma_{B\otimes A}, \Gamma_B\otimes\Gamma_A, y)(\Gamma_{A\otimes B}, \Gamma_{B\otimes A}, c_{A,B}) = (\Gamma_{A\otimes B}, \Gamma_B\otimes\Gamma_A, yc_{A,B}).$$

Since y is $id_{B\otimes A}$ or $c_{B,A}$, then $x = yc_{A,B}$ is $c_{A,B}$ or $id_{A\otimes B}$. Therefore

$$\breve{\Gamma}_{B,A}\Gamma(c_{A,B}) = (\Gamma_{A\otimes B}, \Gamma_B\otimes\Gamma_A, yc_{A,B}) = \breve{\Gamma}_{A,B};$$

i.e the following diagram is commutative

$$\begin{array}{ccc}
\Gamma(A\otimes B) & \xrightarrow{\breve{\Gamma}_{A,B}} & \Gamma A\otimes\Gamma B \\
\downarrow{\scriptstyle \Gamma(c_{A,B})} & & \| \\
\Gamma(B\otimes A) & \xrightarrow{\breve{\Gamma}_{B,A}} & \Gamma B\otimes\Gamma A
\end{array}$$

Since $\Gamma(\underline{1}) = \Gamma_1$ is an unit object of $\underline{M(C)}$, the $\otimes$-functor $(\Gamma, \breve{\Gamma})$ is compatible with the unity constraints (Chi I, §4, n°2, prop. 8, [2]).

By definition of Γ we immediately see that it is a full representative, faithfull functor because the corespondance $u \mapsto \Gamma(u)$ is a bijection from $Hom_{\underline{C}}(A,B)$ to $Hom_{\underline{M\langle C\rangle}}(\Gamma A, \Gamma B)$ and every $F\in Ob\underline{M\langle C\rangle}$ is in the form $F=\Gamma_{A_1}\otimes\ldots\otimes\Gamma_{A_n}$, hence $F \simeq \Gamma(A_1\otimes\ldots\otimes A_n)$.

Thus, $(\Gamma, \breve{\Gamma})$ is a $\otimes$-ACU equivalence and the theorem is proved.

Combining this theorem and theorem 2.7 [6] we obtain:

Thoerem 2.2 Every $\otimes$-ACU category A in which the commutativity constraint c satisfies the condition (0.1), i.e $c_{A,A} = id_{A\otimes A}$, for all $A\in Ob\underline{A}$, is $\otimes$-ACU equivalent to a $\otimes$-strict ACU category

Poorf. Suppose that A is a $\otimes$-ACU category with conditioh (0. 1). Then by theorem 2.7 [6] there exists a $\otimes$-ACU equivalence

$$(\Phi, \breve{\Phi}) : \underline{A} \to \underline{End(A_d)}.$$

in which $\underline{End}(A_d)$ is a $\otimes$-ACU category and a $\otimes$-strict AU category. Furthermore, $\underline{End}(A_d)$ satisfies also the condition (0. 1). In fact, by proposition 8, §5. Ch. I, [2], for $(F, \breve{F}) \in Ob\ \underline{End}(A_d)$, $c_{(F,\breve{F}),(F,\breve{F})}$ is defined by the following commutative diagram:

$$\begin{array}{ccc} \Psi((F,\breve{F}) \otimes (F,\breve{F})) & \xrightarrow{\breve{\Psi}_{(F,\breve{F}),(F,\breve{F})}} & \Psi(F,\breve{F}) \otimes \Psi(F,\breve{F}) \\ \Psi(c_{(F,\breve{F}),(F,\breve{F})}) \downarrow & & \| \; c_{F1,F1} = id_{F1 \otimes F1} \\ \Psi((F,\breve{F}) \otimes (F,\breve{F})) & \xrightarrow{\breve{\Psi}_{(F,\breve{F}),(F,\breve{F})}} & \Psi(F,\breve{F}) \otimes (F,\breve{F}) \end{array}$$

It follows immediately:

$$c_{(F,\breve{F}),(F,\breve{F})} = id_{(F,\breve{F}) \otimes (F,\breve{F})}.$$

Thus, by theorem 2.1, we have:

$$(\Gamma, \breve{\Gamma}) : \underline{End}(A_d) \xrightarrow{\approx} \underline{M}\langle \underline{End}(A_d) \rangle.$$

Finally, we get a $\otimes$-ACU equivalence

$$(\Gamma\Phi, \breve{\Gamma\Phi}) : \underline{A} \xrightarrow{\approx} \underline{M}\ \underline{End}(A_d)$$

and this establishes the theorem.

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