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ON COMPLEX ANALYSIS IN VIETNAM

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Abstract. In the development of contemporary mathematics in Vietnam complex analysis occupies a special place. We give a brief survey of the development of complex analysis in Vietnam since 1947, when the first mathematical research paper (and it was on complex analysis) written by a Vietnamese mathematician was published in an international journal. We describe how complex analysis in Vietnam developed under very special conditions: the anti-French resistance, the struggle for the reunification of the country, the American war, the economic crisis, and the change toward a market economy.

In the development of contemporary mathematics in Vietnam complex analysis occupies a special place. In this note we give a brief survey of the development of complex analysis in Vietnam. We describe how complex analysis in Vietnam developed under very special conditions: the anti-French resistance, the struggle for the reunification of the country, the American war, the economic crisis, and the change toward a market economy.

1. Le Van Thiem - the founder of complex analysis in Vietnam

The history of complex analysis, as well as of the contemporary mathematics in Vietnam, dates from back to 1947, when a Vietnamese mathematician, Le Van Thiem, published a paper in an international journal (*Beitrag zum Typenproblem der Riemannschen Flachen*; Commentarii Mathematici Helvertici, 20, 1947, pp. 270-287).

Le Van Thiem was born in 1918 in Ha Tinh, Vietnam, into an intellectual family. He was the youngest from 13 brothers and sisters. The oldest brother of Le Van Thiem got the "doctor" degree (Tien si) in the last Confucian tradition examination (1919, Nguyen Dynasty), while Le Van Thiem was the first Vietnamese who get the "modern" doctor degree.

In 1939, after passing the final term examination excellently, Le Van Thiem was offered a scholarship to study at the École Normale Supérieure in Paris. His education was interrupted by the outbreak of the Second World War, and only continued in 1941. He graduated with Bachelor Degree in Mathematics within a year rather than the conventional 3-year time. In 1942 under the supervision of

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George Valiron he began his research on the value distribution theory of meromorphic functions (Nevanlinna Theory). It was in this period that he made important contributions to the solution of the inverse problem of Nevanlinna theory that contributed the core of his doctoral thesis (1945, Goettingen) and Docteur d'État (1949, Paris), and placed him among the best young researchers in the field at that time [1]. Meanwhile, in Vietnam, the resistance war against French colonialists was at its heights. Despite his great passion for mathematics and the bright prospect of his scientific career, in 1949 Le Van Thiem took a dramatic decision which would not only drastically change his life but which would exert a profound influence on many generations of students in Vietnam to come - abandoning his academic position at the prestigious Zurich University, he returned to Vietnam to actively take part in Vietnam's struggle for independence.

To return to Vietnam, Le Van Thiem first flew to Bangkok and then headed for the liberated region in the far south of Vietnam. A few months later, following a narrow footpath through the mountains, which later during the American war became the famous Ho Chi Minh trail, he made the long trek to Viet Bac, in the far north of Vietnam and which used to be the headquarters of the Resistance. It was in Viet Bac that Le Van Thiem met other intellectuals, most of them educated in France - Ta Quang Buu (a mathematician, former Minister of Defense (1947), Minister of Higher Education, and President of the National Committee of Science and Technology), Tran Dai Nghia (a former *polytechnicien* and President of the Vietnamese Academy of Science). Convinced of the importance of education and science in this fight, Le Van Thiem founded, in the liberated zone, a teacher training college and a college of fundamental sciences with the aim of providing the country with qualified teachers and technicians, of which the resistance was in dire need. These colleges functioned until the end of the French war in 1954. The later development of science and research in Vietnam highlighted the essential contribution of these colleges to upgrading and sustaining the education system at a satisfactory level, even in complete isolation from the outside world during the French and then the American wars. Furthermore, these colleges formed the foundation for the immediate reopening in 1955 of Hanoi University with a strictly Vietnamese teaching staff, which at that time was a remarkable accomplishment in this region of Asia. Le Van Thiem, together with other mathematicians (Hoang Tuy, Ta Quang Buu) founded two Vietnamese research mathematical journals in foreign languages (English, French, and Russian): *Acta Mathematica Vietnamica* and the *Vietnam Journal of Mathematics*. He also was a founder of the journal *Mathematics and Youth*, a friend of many generations of secondary school students. The appearance of these three journals during the American war in Vietnam was an important and hardly believable event.

[1] See: D. Drasin. *A meromorphic function with assigned Nevanlinna deficiencies.* Bulletin of the American Mathematical Society, Vol. 80, N. 4, July 1974. In this paper Drasin commented on Le Van Thiem's paper: *Using an important principle of Teichmuller, Le Van Thiem first applied this principle to the inverse problem, and the method was further exploited by Goldberg.*

Le Van Thiem passed away on June 3rd, 1991 in Ho Chi Minh city. He was the first modern Vietnamese mathematician to be commemorated by having a street (located in Hanoi) named after him.

2. The period of 1954-1975

The resistance war was finished in 1954, and the university reopened in Hanoi in 1955. Le Van Thiem was the rector of the university. In the first years Le Van Thiem continued to study his old subjects: classification of Riemann surfaces. However, following the scientific policy of the government to promote applied sciences, Le Van Thiem began to study the theory of groundwater movement and its applications in Vietnam. And in this new area for him, Le Van Thiem obtained a remarkable result: he is the first who solved explicitly the problem of filtration via two ground layers, by using the Riemann-Schwarz symetry principle in complex analysis.[2]

In 1964 the U. S. army began to bombard the North Vietnam, including Hanoi and other cities. All the universities have been evacuated to the forest, many of them were moved to Viet Bac, the former headquarter of the anti-French resistance. However, even during the war, the Vietnamese mathematical community continued its activities.

The mathematical society, which was founded in 1965 by Le Van Thiem, organized joint seminars on optimization, probability, functional analysis, complex analysis, algebra and numerical analysis. People from the Mathematics Unit of the National Committee of Science and Technology (the precursor of the Institute of Mathematics), the Hanoi University, the Pedagogical Institute, and the Polytechnic Institute participated. Since these Institutions had been evacuated in different directions from Hanoi, the seminars were held in Hanoi. They meet twice a month. One should say that people were very diligent about attending. In the seminar on complex analysis the main topics included applications of the theory of holomorphic functions in the problems of orienting explosives, groundwater movement and desalt of the coastal area.

The first group on complex analysis consisted of the students of Le Van Thiem at the Hanoi University: Ngo Van Luoc, Hoang Dinh Dung, Nguyen Van Lam, Vo Dang Thao, Nguyen Van Khue, Le Van Thanh, Nguyen Dinh Sang, Ha Huy Khoai, Le Hung Son. In the years 1968-1970 some people of the group received grants to follow postgraduate programs abroad: Ngo Van Luoc (Georgia, generalized analytic functions, under supervision of Vekua), Hoang Dinh Dung (Bielorussia, boudary problems of holomorphic functions, under supervision of Gakhov), Nguyen Van Lam, Vo Dang Thao (Romania, quasi-conformal mappings, under supervision of Cazacu).

In the years 1966-1969 Le Van Thiem and his students (Ngo Van Luoc, Le Van Thanh, Ha Huy Khoai, Le Hung Son and others) have applied the method of complex analysis in orienting explosives during the time of the American war.

[2]See: P. Ya. Palubarinova-Kochina. *Theory of Groundwater Movements*, Nauka, Moscow 1977 (in Russian).

The idea is quite simple: under a great pressure of an explosion the materials around the explosion center are moving as an ideal liquid. It is well-known that the moving of an ideal liquid can be described by a holomorphic function. By using the Christoffel-Schwarz formula to map the considered domain into the above half plane we can find this holomorphic function, and then control the explosion. The students of the group "complex analysis" of the Hanoi University have applied the method of orienting explosives to dredge the Le Dynasty canal, which played an important role during the war, and to construct a strategic road in the jungle.

When the American war was going to its end, young people of the complex analysis group would like to learn some "morden" topics in complex analysis[3]. In 1970 a learning seminar "Several complex variables" was created by Ha Huy Khoai, Nguyen Van Khue and Le Van Thanh. At that time in Vietnam nobody had have any background on several complex variables. The participants of the seminar randomly found two books, translated into Russian: Gunning and Rossi: "Introduction to several complex variables" and M. Hervé: "Locally analytic sets", and these books became the first materials of the seminar. After one year Ha Huy Khoai and Nguyen Van Khue published the first papers on complex manifolds in *Acta Math. Vietnam* and *Vietnam J. Math*, and submited a paper to the russian journal *Funct. Anal. and its Appl.*, which was published in 1973. The first group on several complex variables was founded, with Ha Huy Khoai, Nguyen Van Khue, Nguyen Dinh Sang, Le Hung Son, and Le Van Thanh.

3. The period of 1975-1995

After the reunification of the country in 1975 the mathematics in Vietnam had the favorable conditions for development. Especially, the cooperation with the mathematical community in the world became much easier. Many young people obtained fellowships to go study abroad, and not only to the socialist countries, but also to other countries: France, West Germany, Italy, Japan, etc. When people with Ph. D degree retuned from abroad, some groups established in the Institute of Mathematics, the Hanoi University, the Hanoi Pedagogical Institute, the Hanoi Polytechnic Institute. Various topics of complex analysis are interested: generalized analytic functions (Ngo Van Luoc, Le Hung Son), boundary problems of analytic functions and singular integral equations (Hoang Dinh Dung, Nguyen Van Mau), quasiconformal mappings (Nguyen Van Lam, Vo Dang Thao), several complex variables (Nguyen Van Khue, Le Mau Hai, Do Duc Thai, Nguyen Dinh Sang, Le Van Thanh).

However, during the period from 1980 to 1995, mathematics in Vietnam faced a serious difficulty. Vietnam experienced an economic crisis during the 1980's, and in the beginning of the 1990's, Vietnam began its transition toward a market

[3]For them, it was difficult to know what is "morden", as Grothendieck wrote in his famous report on his visit to Vietnam in 1967:"In a country which, by force of circumstance, has few relations with the outside (unless one counts the cluster bombs as a form of relation), it is particularly difficult for an inexperienced mathematician to orient himself among the multitude of possible directions, to distinguish what is interesting from what is not."

economy, the so called *Doi moi*. Many mathematicians had to leave mathematics because the salary of a mathematics lecturer was very low, only about 3–4 USD per month. Almost all had to do a "second job", which usually required much more time and effort than the first one – doing mathematics! If, for many years, mathematics was the first choice for the best high school students, then in the early 90's, an opposite tendency appeared. It even happened one year that there was not a single student who entered the mathematics department of Hanoi University. At that time, some mathematicians predicted that mathematics in Vietnam was at risk to become extinct in only 15 years! [4]

Fortunately, mathematics in Vietnam survived this difficult period. The first and the most important reason was that during this period many Vietnamese mathematicians continued their mathematical research despite the extremely hard conditions. On the other hand, it is worth mentioning valuable help from the mathematical community around the world, especially from France, Italy, Germany, and Japan. I would like to mention here the role of the program "ForMathVietnam" from France and the importance of fellowships like Alexander-von-Humboldt (Germany), JSPS (Japan), and ICTP (Italy and UNESCO).

4. The period of 1995-present

Beginning in the mid-1990's, Vietnam step-by-step got out of the economic crisis, and Vietnamese mathematics returned again to a normal development. Young Vietnamese now can go to study abroad not only with fellowships from foreign institutions, but also with financial support from the Vietnamese government (Project 322). Good students with a passion for mathematical research now do not hesitate to choose mathematics as their future career. In this circumstance complex analysis in Vietnam made a remarkable progress. Established some groups on complex analysis, which publish regularly research papers in international journals: the group on potential theory at the Hanoi National University of Education (Nguyen Van Khue, Le Mau Hai, Nguyen Quang Dieu and others), the group on complex geometry and Nevanlinna theory at the Hanoi National University of Education (Do Duc Thai, Tran Van Tan, Si Duc Quang and others), the group on Nevanlinna theory and Number theory at the Institute of Mathematics (Ha Huy Khoai, Ta Thi Hoai An and others). At the present time at the departments of mathematics of several universities there are people working in complex analysis.

In the development of complex analysis in Vietnam the cooperation with foreign colleagues plays an important role. Young Vietnamese are offered fellowships for studying in the Europe (France, Germany, Italy, Sweden), Japan, Taiwan, and other countries, while other established mathematicians obtain opportunities to work with their colleagues in abroad. In recent years many international conferences and schools on complex analysis and complex geometry are organized in Vietnam, with the participation of leading experts in complex analysis and related domains from around the world.

Complex analysis, the first domain of contemporary mathematics in Vietnam, remains to be one of the strongest research fields of mathematics in Vietnam.

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