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nX-COMPLEMENTARY GENERATIONS OF THE SPORADIC GROUP Co_1

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Abstract. A finite group G with conjugacy class nX is said to be nX-complementary generated if, for any arbitrary $x \in G - \{1\}$, there is an element $y \in nX$ such that $G = \langle x, y \rangle$. In this paper we prove that the Conway's group Co_1 is nX-complementary generated for all $n \in \Pi_e(Co_1)$. Here $\Pi_e(Co_1)$ denotes the set of all element orders of the group Co_1.

1. Introduction

Let G be a group and nX a conjugacy class of elements of order n in G. Following Woldar [25], the group G is said to be nX-complementary generated if, for any arbitrary non-identity element $x \in G$, there exists a $y \in nX$ such that $G = \langle x, y \rangle$. The element $y = y(x)$ for which $G = \langle x, y \rangle$ is called complementary. In [25], Woldar proved that every sporadic simple group is pX-complementary generated for the greatest prime divisor p of the order of the group.

A group G is said to be (lX, mY, nZ)-generated (or (l, m, n)-generated for short) if there exist $x \in lX$, $y \in mY$ and $z \in nZ$ such that $xy = z$ and $G = \langle x, y \rangle$. As a consequence of a result in [25], a group G is nX-complementary generated if and only if G is (pY, nX, t_pZ)-generated for all conjugacy classes pY with representatives of prime order and some conjugacy class t_pZ (depending on pY).

Suppose G is a finite group and $\Delta(G) = \Delta(lX, mY, nZ)$ denotes the structure constant of G for the conjugacy classes lX, mY and nZ. It is a well-known fact that this value is the cardinality of the set

$$\Gamma = \{(x, y) \in lX \times mY | xy = z\},$$

where z is a fixed element of the class nZ. Define $\Delta^*(G) = \Delta^*_G(lX, mY, nZ)$ as the number of pairs $(x, y) \in \Gamma$ such that $G = \langle x, y \rangle$. It is clear that if $\Delta^*(G) > 0$ then G is (lX, mY, nZ)-generated. In this case, (lX, mY, nZ) is called a generating triple for G. It is easy to see that if $\pi \in S_3$ and G is (l, m, n)-generated then it is $((l)\pi, (m)\pi, (n)\pi)$-generated. Finally, for subgroups $H_1, \cdots, H_r$ of G, $\Sigma(H_1 \cup \cdots \cup H_r)$ denotes the number of pairs $(x, y) \in \Gamma$ such that $\langle x, y \rangle \leq H_i$ for some $1 \leq i \leq r$.

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In a series of papers, [15-20] Moori and Ganief established all possible (p, q, r)-generations and nX-complementary generations, where p, q, r are distinct primes of the sporadic groups J_1, J_2, J_3, HS, McL, Co_3, Co_2, and F_{22}. Also, the authors of [2-3] and [8-14] did the same for the sporadic groups Co_1, $O'N$, Ly, Suz, Ru and He. The motivation for this study is outlined in these papers and the reader is encouraged to consult these papers for background material as well as basic computational techniques.

We shall use the ATLAS notation [6] for conjugacy classes, character tables, maximal subgroups and etc. Computations were carried out with the aid of GAP [21]. See [7], [15], [23], [24] and [25] for discussion and background material about the maximal subgroups of the group Co_1, (p, q, r)-generations and nX-complementary generations of finite groups.

In this paper our notation is standard and taken mainly from [1], [15] and [16]. We will prove the following result:

Theorem 1.1. *The Conway group Co_1 is nX-complementary generated if and only if $n \geq 4$ and $nX \notin \{4A, 4B, 4C, 4D, 5A, 6A\}$.*

2. Preliminaries

In this section, we discuss techniques that are useful in resolving generation type questions for finite groups. We begin with a theorems of Scott that, in certain situations, is very effective at establishing non-generations (see [22]).

Theorem 2.1. (Scott's Theorem, [22]). *Let $x_1, x_2, \ldots, x_m$ be elements generating a group G with $x_1 x_2 \cdots x_m = 1$, and let V be an irreducible module for G of dimension n. Let $C_V(x_i)$ denote the fixed point space of $\langle x_i \rangle$ on V, and let d_i be the dimension of $V/C_V(x_i)$. Then $d_1 + \cdots + d_m \geq 2n$.*

Further useful results that we shall use are:

Lemma 2.1. (Conder, [5]). *Let G be a finite centerless group and suppose lX, mY and nZ are G-conjugacy classes for which*

$$\Delta^\star(G) = \Delta^\star_G(lX, mY, nZ) < |C_G(z)|, z \in nZ.$$

Then $\Delta^\star(G) = 0$ and therefore G is not (lX, mY, nZ)-generated.

Lemma 2.2. (Ganief and Moori, [16]). *Let G be a finite group and H a subgroup of G containing a fixed element x such that*

$$\gcd(o(x), [N_G(H) : H]) = 1.$$

Then the number h of conjugates of H containing x is $\chi_H(x)$, where $\chi_H(x)$ is the permutation character of G with action on the conjugates of H. In particular, $h = \sum_{i=1}^{m} \frac{|C_G(x)|}{|C_{N_G(H)}(x_i)|}$, where $x_1, \ldots, x_m$ are representatives of the $N_G(H)$-conjugacy classes that fuse to the G-conjugacy class of x.

Lemma 2.3. (Ganief and Moori, [17]). *Let G be a $(2X, sY, tZ)$-generated simple group then G is $(sY, sY, (tZ)^2)$-generated.*

Lemma 2.4. (Ganief and Moori, [18]). *If G is nX-complementary generated and $(sY)^k = nX$, for some integer k, then G is sY-complementary generated.*

For any positive integer n, $T(2,2,n) \cong D_{2n}$, the dihedral group of order $2n$. Thus, if G is a finite group which is not isomorphic to some dihedral group, then G is not $(2X, 2X, nY)$-generated, for all classes of involutions and any G-class nY. Thus, Co_1 is not $2X$-complementary generated. Also, by a result of Woldar [25], mentioned in Section 1, the group Co_1 is $23X$-complementary generated for $X \in \{A, B\}$.

Authors in [8] and [9] proved the following theorems which will be used later.

Theorem 2.2. *The group Co_1 is (pX, qY, t_pZ)-generated for each prime class pX and $q \in \{7, 11, 13, 23\}$.*

Theorem 2.3. *The group Co_1 is $(pX, 5Y, t_pZ)$-generated for each prime class pX and $Y \in \{B, C\}$.*

3. nX-COMPLEMENTARY GENERATION OF THE GROUP Co_1

In this section we investigate the nX-complementary generations of the group Co_1. At the end of Section 2, we show that the Conway group Co_1 is not $2X$-complementary generated. In the following simple lemma, we investigate the $3X$-complementary generation of the group Co_1.

Lemma 3.1. *The group Co_1 is not $3X$-complementary generated.*

Proof. Suppose $X = A$. With a tedious computation, we can see that for all triples of the form $(5A, 3A, tZ)$,

$$\Delta^{\star}_{Co_1}(5A, 3A, tZ) < |C_{Co_1}(tZ)|.$$

Hence, by Lemma 2.1, $\Delta^{\star}(Co_1) = 0$ and so Co_1 is not $(5A, 3A, tX)$-generated. Next, we assume that $Y \in \{B, C, D\}$. Again, we can see that

$$\Delta^{\star}_{Co_1}(2A, 3Y, tZ) < |C_{Co_1}(tZ)|,$$

and so the group Co_1 is not $(2A, 3Y, tX)$-generated, for $Y \in \{B, C, D\}$. These computations show that the group Co_1 is not $3X$-complementary generated. □

Lemma 3.2. *The group Co_1 is $4X$-complementary generated if and only if $X \in \{E, F\}$.*

Proof. Set $K = \{A, B, C, D\}$. First of all, we consider the conjugacy class $2A$. Then we have

$$\Delta_{Co_1}(2A, 4X, tZ) < |C_{Co_1}(tZ)|,$$

in which $X \in \{A, B\}$ and tZ is an arbitrary conjugacy class. This shows that Co_1 is not $4A-$ and $4B-$complementary generated. Choose the conjugacy class $3A$. We can see that

$$\Delta_{Co_1}(3A, 4D, tZ) < |C_{Co_1}(tZ)|,$$

for all conjugacy class tZ of Co_1. Thus, Co_1 is not $4D-$complementary generated. We now show that Co_1 is not $4C-$complementary generated. To do this, we consider the conjugacy class $2A$. Suppose

$$M = \{7B, 8E, 9C, 10D, 10E, 10F, 11A, 12H, 12I, 12K, 14B, 15D, 15E, \\ 16A, 16B, 18C, 20C, 21C, 22A, 23A, 23B, 24E, 24F, 28A, 30D, 30E\}.$$

If $tZ \notin M$ then

$$\Delta_{Co_1}(2A, 4C, tZ) < |C_{Co_1}(tZ)|,$$

and, by Lemma 2.1, $\Delta^{\star}_{Co_1}(2A, 4C, tZ) = 0$.

On the other hand, the group Co_1 acts on a 24-dimensional vector space $\bar{\Lambda}$ over $GF(2)$ and has three orbits on the set of non-zero vectors. The stabilizers are the groups Co_2, Co_3 and $2^{11} : M_{24}$, and the permutation characters of Co_1 on $\bar{\Lambda} - \{0\}$ is

$$\chi = 1_{Co_2} \uparrow^{Co_1} + 1_{Co_3} \uparrow^{Co_1} + 1_{2^{11}:M_{24}} \uparrow^{Co_1},$$

and using GAP [10], we find that

$$\chi = 3 \times 1a + 2 \times 299a + 3 \times 17250a + 3 \times 80730a + 376740a + \\ + 644644a + 2055625a + 2417415a + 2 \times 5494125a,$$

where na denotes the first irreducible character with degree n, as in ATLAS (see [6]). Now for $g \in Co_1$, the value of $\chi(g)$ is the number of non-zero vectors of $\bar{\Lambda}$ fixed by g and hence

$$\chi(g) + 1 = |C_V(g)| = 2^{\delta_{nX}},$$

where $g \in nX$ and δ_{nX} is the dimension of the fixed space $C_V(g)$. Using the character table of Co_1 we list in Table IV, the values of $d_{pX} = 24 - \delta_{pX}$ for all conjugacy classes of Co_1.

We now consider the generation of $(2A, 4C, tZ)$, for $tZ \in M$. Using Table IV, we have,

$$d_{2A} + d_{4C} + d_{tZ} = 8 + 14 + d_{tZ} < 48$$

and hence, by Scott's theorem $(2A, 4C, tZ)$ is a non-generating triple of Co_1. We now investigate the $4E$- and $4F$-complementary generations of the group Co_1. Our main proof will consider two cases.

Co_1 *is* $4E$*-complementary generated.* First, we assume that $pY = 3A$. Choose the conjugacy class $t_pZ = 36A$. The only maximal subgroups of Co_1 that may contain $(3A, 4E, 36A)$-generated proper subgroups are isomorphic to $2^{1+8}_{+} \cdot O^{+}_{8}(2)$ and $3^{3+4} : 2(S_4 \times S_4)$. Moreover,

$$\Delta(Co_1) = 252, \quad \Sigma(2^{1+8}_{+} \cdot O^{+}_{8}(2)) = 36, \quad \Sigma(3^{3+4} : 2(S_4 \times S_4)) = 0.$$

On the other hand,

$$\Delta^{\star}(Co_1) \geq 252 - 36 - 0 > 0$$

and so Co_1 is $(3A, 4E, 36A)$-generated. We next suppose that $pY \neq 3A$. Choose the conjugacy class $39A$. The maximal subgroups of Co_1 with non-empty intersection with the conjugacy classes $39A$ and $4E$ are, up to isomorphisms, $3 \cdot Suz \cdot 2$ and $(A_4 \times G_2(4)) : 2$. In Table III, we calculate $\Delta(Co_1)$, $\Sigma(3 \cdot Suz \cdot 2)$ and $\Sigma((A_4 \times G_2(4)) : 2)$. By this table, we can see that for any prime class pX, $\Delta^{\star}_{Co_1}(pX, 4E, 39A) > 0$. Thus, Co_1 is $4E$-complementary generated.

Co_1 is $4F$-complementary generated. From the list of maximal subgroups of Co_1 we observe that, up to isomorphisms, $(A_4 \times G_2(4)) : 2$ is the only maximal subgroup of Co_1 with non-empty intersection with the conjugacy classes $4F$ and $39A$. By Table III, we can see that for any prime class pX, $\Delta^{\star}_{Co_1}(pX, 4F, 39A) > 0$. Thus, Co_1 is $4F$-complementary generated. This completes the proof. □

Lemma 3.3. *The group Co_1 is not $5A$-complementary generated, but it is $5B$- and $5C$-complementary generated.*

Proof. Consider the conjugacy class $3A$. Then for all conjugacy class tZ, we have

$$\Delta_{Co_1}(3A, 5A, tZ) < |C_{Co_1}(tZ)|.$$

This shows that Co_1 is not $5A$-complementary generated. The cases $5B$ and $5C$ follow from Theorems 2.2 and 2.3. □

Lemma 3.4. *The group Co_1 is pX-complementary generated for each prime divisor $p \geq 7$.*

Proof. The result follows from Theorems 2.2 and 2.3. □

Lemma 3.5. *The group Co_1 is $6X$-complementary generated if and only if $X \neq A$.*

Proof. Consider the conjugacy class $2A$. We can see that for all conjugacy class tZ,

$$\Delta_{Co_1}(2A, 6A, tX) < |C_{Co_1}(tZ)|.$$

This shows that the group Co_1 is not $6A-$ complementary generated. Thus, it is enough to investigate four cases that which we treat separately.

Case $6B$. We first assume that $pY = 3A$. Choose the conjugacy class $20C$. From the list of maximal subgroups of Co_1 we observe that, up to isomorphisms, $2^{1+8}_{+} \cdot O^{+}_{8}(2)$ and $D_{10} \times ((A_5 \times A_8).2).2$ are the only maximal subgroups of Co_1 that admit $(3A, 6B, 20C)$-generated subgroups. From the structure constant we calculate

$$\Delta(Co_1) = 20, \quad \Sigma(2^{1+8}_{+} \cdot O^{+}_{8}(2)) = \Sigma(D_{10} \times ((A_5 \times A_8).2).2) = 0.$$

Thus, $\Delta^{\star}(Co_1) > 0$. We next suppose that $pY \neq 3A$. Choose the conjugacy class $39A$. The maximal subgroups of Co_1 with non-empty intersection with the conjugacy classes $39A$ and $6B$ are, up to isomorphisms, $3 \cdot Suz \cdot 2$ and $(A_4 \times G_2(4)) : 2$. In Table V, we calculate $\Delta(Co_1)$, $\Sigma(3 \cdot Suz \cdot 2)$ and $\Sigma((A_4 \times G_2(4)) : 2)$. By this table, we can see that for any prime class pX, $\Delta^{\star}_{Co_1}(pX, 6B, 39A) > 0$. This shows that the group Co_1 is $6B$-complementary generated.

Case $6C$. Suppose $pY = 3A$. Choose the conjugacy class $15E$. Among the maximal subgroups of Co_1 with order divisible by 30, the only subgroups with empty intersection with any conjugacy class in this triple are isomorphic to $2_+^{1+8} \cdot O_8^+(2)$ and $3^{1+4} \cdot 2U_4(2) \cdot 2$. We can see that

$$\Delta(Co_1) = 120, \quad \Sigma(2_+^{1+8} \cdot O_8^+(2)) = \Sigma(3^{1+4} \cdot 2U_4(2) \cdot 2) = 0.$$

Our calculations give, $\Delta^\star(Co_1) \geq 0$. On the other hand, using Table V and similar argument as in above, we can see that the group Co_1 is $6C$-complementary generated.

Case $6D$. Suppose $pY = 3A$. Choose the conjugacy class $16B$. The only maximal subgroups of Co_1 that may contain $(3A, 6D, 16B)$-generated subgroups, are isomorphic to $2_+^{1+8} \cdot O_8^+(2)$ and $2^{4+12} \cdot (S_3 \times 3S_6)$. We easily calculate the structure constants

$$\Delta(Co_1) = 160, \quad \Sigma(2_+^{1+8} \cdot O_8^+(2)) = 32, \quad \Sigma(2^{4+12} \cdot (S_3 \times 3S_6)) = 0.$$

Thus, $\Delta^\star(Co_1) > 0$. On the other hand, using Table V and similar argument as in above, we can see that the group Co_1 is $6D$-complementary generated.

Case $6Y$*,* $Y \in \{E, F, G, H, I\}$. We claim that Co_1 is $(pX, 6Y, 39A)$-generated for $Y \in \{E, F, G, H, I\}$ and each prime class pX. To do this, the only maximal subgroup of Co_1, up to isomorphisms, with non-empty intersection with any conjugacy class in above triples are $3.Suz.2$ and $(A_4 \times G_2(4)) : 2$. In Table V, we calculate $\Delta^*_{Co_1}(pX, 6Y, 39A)$, for each prime class pX and $Y \in \{E, F, G, H, I\}$. From the calculation of this table, we can see that $\Delta^*(G) > 0$, for each triple $(pX, 6Y, t_pZ)$, $X \in \{E, F, G, H, I\}$. Therefore, the group Co_1 is $6X$-complementary generated for $X \in \{E, F, G, H, I\}$. This completes the proof. □

Lemma 3.6. *The group* Co_1 *is* $8X$*-complementary generated for each* $X \in \{A, B, C, D, E, F\}$.

Proof. In Table V, we calculate $\Delta^*_{Co_1}(pX, 8Y, 39A)$, for each prime class pX and $Y \in \{A, B, C, D, F\}$. By this table, $\Delta^*(Co_1)(pY, 8X, t_pZ) > 0$ for each triples. Hence the group Co_1 is $8X$-complementary generated, for all $X \in \{A, B, C, D, F\}$. On the other hand, there is no maximal subgroup of the group Co_1 which intersects the conjugacy classes $8E$ and $39A$. Since $\Delta(pX, 8E, 39A) > 0$, for all prime class pX, Co_1 is $8E$-complementary generated, proving the result. □

Lemma 3.7. *The group* Co_1 *is* $9X$*-complementary generated for* $X \in \{A, B, C\}$.

Proof. In Table V, we calculate $\Delta^*_{Co_1}(pX, 9Y, 39A)$, for each prime class pX and $Y \in \{A, B, C\}$. Since for each triples, $\Delta^\star(Co_1) > 0$, hence the group Co_1 is $9A$-, $9B$- and $9C$-complementary generated. □

Lemma 3.8. *For all conjugacy class* $10X$*, the group* Co_1 *is* $10X$*-complementary generated.*

Proof. Consider the conjugacy class $39A$. In Table V, we calculate $\Delta^*_{Co_1}$ $(pY, 10X, 39A)$, for each prime class pY and $X \in \{A, B\}$. Since for each triple, $\Delta^*(Co_1) > 0$, hence the group Co_1 is $10A$- and $10B$-complementary generated. On the other hand,

$$(10C)^2 = (10D)^2 = (10F)^2 = 5B$$

and $(10E)^2 = 5C$. The result now follows from Lemmas 2.4 and 3.3. □

Lemma 3.9. *For all conjugacy class $12X$, the group Co_1 is $12X-$ complementary generated.*

Proof. In Table V, we calculate $\Delta^*(Co_1)(pY, 12X, 39A)$, for any prime class pY and every conjugacy class $12X$, $X \in \{A, B, C\}$. For $X \in \{B, C\}$ and each prime class pY, $\Delta^*_{Co_1}(pY, 12X, 39A) > 0$. Hence the group Co_1 is $12X$-complementary generated, $X \in \{B, C\}$. For $X = A$ and $pY \neq 3A$, again consider the conjugacy class $39A$. In this case, by Table V, $\Delta^*_{Co_1}(pY, 12A, 39A) > 0$. For the conjugacy class $pY = 3A$, we assume that $t_pZ = 33A$. Then we can see that the only maximal subgroups of Co_1 that may contain $(3A, 12A, 33A)$-generated subgroups, are isomorphic to $3 \cdot Suz \cdot 2$ and $U_6(2) \cdot 3 \cdot 2$. We easily calculate the structure constant $\Delta(Co_1) = 33$ and

$$\Sigma(3 \cdot Suz \cdot 2) = \Sigma(U_6(2) \cdot 3 \cdot 2) = 0.$$

We now consider the conjugacy classes $12X, X \notin \{A, B, C\}$. We can see that

$$(12D)^2 = (12K)^2 = 6F, \quad (12F)^3 = 4E,$$

$$(12E)^2 = (12G)^2 = (12I)^2 = (12J)^2 = 6E, \quad (12H)^2 = 6D, \quad (12L)^2 = 6H$$

and $(12M)^2 = 6I$. The result now follows from Lemmas 2.4, 3.2 and 3.5. □

Lemma 3.10. *The group Co_1 is $15X$-complementary generated for*

$$X \in \{A, B, C, D, E\}.$$

Proof. From the character table of Co_1 [6] we can see that

$$(15B)^3 = (15D)^3 = 5B, \quad (15E)^3 = 5C.$$

Hence by Lemma 2.4, Co_1 is $15X$-complementary generated, for $X \in \{B, D, E\}$. Suppose $X \in \{A, C\}$. Then we can see that the maximal subgroups of Co_1 with non-empty intersection with the conjugacy classes $39A$ and $15X$, $X = A$ or C, are, up to isomorphisms, $3 \cdot Suz \cdot 2$ and $(A_4 \times G_2(4)) : 2$. In Table V, we calculate $\Delta(Co_1)$, $\Sigma(3 \cdot Suz \cdot 2)$ and $\Sigma((A_4 \times G_2(4)) : 2)$. By this table, for any prime class pY, $\Delta^\star_{Co_1}(pY, 15X, 39A) > 0$. This shows that the group Co_1 is $15A-$ and $15C-$complementary generated. □

Lemma 3.11. *The group Co_1 is nX-complementary generated for $n = 14$ or $n \geq 16$.*

Proof. The group Co_1 is $14A$-and $14B$-complementary generated because $(14A)^2 = 7A$ and $(14B)^2 = 7B$. The result now follows from the character table of Co_1, Table I and Lemmas 2.3, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, and 3.9. □

Theorem 1.1, the main result of this paper, now follows from Lemmas 3.2-3.11.

Table I
The Power Map of some Conjugacy Classes of Co_1

$(16A)^2 = 8C$	$(16B)^2 = 8D$	$(18A)^3 = 6F$	$(18B)^3 = 6F$
$(18C)^3 = 6F$	$(20A)^2 = 10A$	$(20B)^4 = 5B$	$(20C)^4 = 5C$
$(21A)^3 = 7A$	$(21B)^3 = 7A$	$(21C)^3 = 7B$	$(22A)^2 = 11A$
$(24A)^3 = 8A$	$(24B)^2 = 12B$	$(24C)^3 = 8A$	$(24D)^2 = 12C$
$(24E)^2 = 12E$	$(24F)^2 = 12H$	$(26A)^2 = 13A$	$(28A)^2 = 14B$
$(28B)^2 = 14A$	$(30A)^2 = 15A$	$(30B)^2 = 15B$	$(30C)^2 = 15C$
$(30D)^2 = 15D$	$(30E)^2 = 15E$	$(33A)^3 = 11A$	$(35A)^5 = 7A$
$(36A)^6 = 6F$	$(39A)^3 = 13A$	$(39B)^3 = 13A$	$(40A)^5 = 8A$
$(42A)^6 = 7A$	$(60A)^5 = 12A$		

Table II
The maximal subgroups of Co_1

Group	*Order*	*Group*	*Order*
Co_2	$2^{18}.3^6.5^3.7.11.23$	$3.Suz.2$	$2^{14}.3^8.5^2.7.11.13$
$2^{11}:M_{24}$	$2^{21}.3^3.5.7.11.23$	Co_3	$2^{10}.3^7.5^3.7.11.23$
$2_+^{1+8}.O_8^+(2)$	$2^{21}.3^5.5^2.7$	$U_6(2).S_3$	$2^{16}.3^7.5.7.11$
$(A_4 \times G_2(4)):2$	$2^{15}.3^4.5^2.7.13$	$2^{2+12}:(A_8 \times S_3)$	$2^{21}.3^3.5.7$
$2^{4+12}.(S_3 \times 3S_6)$	$2^{21}.3^4.5$	$3^2.U_4(3).D_8$	$2^{10}.3^8.5.7$
$3^6:2M_{12}$	$2^7.3^9.5.11$	$(A_5 \times J_2):2$	$2^{10}.3^4.5^3.7$
$3^{1+4}.2U_4(2).2$	$2^8.3^9.5$	$(A_6 \times U_3(3)):2$	$2^9.3^5.5.7$
$3^{3+4}:2(S_4 \times S_4)$	$2^7.3^9$	$A_9 \times S_3$	$2^7.3^5.5.7$
$(A_7 \times L_2(7)):2$	$2^7.3^3.5.7^2$	$(D_{10} \times (A_5 \times A_5).2).2$	$2^7.3^2.5^3$
$5^{1+2}:GL_2(5)$	$2^5.3.5^4$	$5^3:(4 \times A_5).2$	$2^5.3.5^4$
$5^2:2A_5$	$2^3.3.5^3$	$7^2:(3 \times 2A_4)$	$2^3.3^2.7^2$

Table III
Calculations of $\Sigma(3 \cdot Suz \cdot 2)$ and $\Sigma((A_4 \times G_2(4)):2))$ for
the Conjugacy Classes $4E$ and $4F$

$(pX, 4E, 36A)$	$2A$	$2B$	$2C$	$3B$
$\Delta(Co_1)$	1248	45276	225888	1536990
$\sum(3 \cdot Suz \cdot 2)$	156	5031	11193	14898
$\sum((A_4 \times G_2(4)):2)$	0	1287	0	0

$(pX, 4E, 36A)$	$3C$	$3D$	$5A$	$5B$
$\Delta(Co_1)$	6820242	157509378	28572336	2408130582
$\sum(3 \cdot Suz \cdot 2)$	29796	480324	288678	1732539
$\sum((A_4 \times G_2(4)):2)$	-	0	0	0

$(pX, 4E, 36A)$	$5C$	$7A$	$7B$	$11A$
$\Delta(Co_1)$	5730636834	4873002810	73302549450	1302907311810
$\sum(3 \cdot Suz \cdot 2)$	-	6177990	-	47145150
$\sum((A_4 \times G_2(4)):2)$	-	0	-	-

Table III (Continued)

$(pX, 4E, 36A)$	$13A$	$23A$		
$\Delta(Co_1)$	550750135026	3736138284216		
$\sum(3 \cdot Suz \cdot 2)$	79868568	-		
$\sum((A_4 \times G_2(4)) : 2)$	2824185	-		

$(pX, 4F, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	1950	128076	712686	234
$\sum((A_4 \times G_2(4)) : 2)$	0	1092	0	0
$(pX, 4F, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	4465968	21585408	495276288	93104544
$\sum((A_4 \times G_2(4)) : 2)$	0	-	0	0
$(pX, 4F, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	7453075968	18100083456	15535263744	229491215616
$\sum((A_4 \times G_2(4)) : 2)$	0	-	0	-
$(pX, 4F, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	4098513988608	1737707392512	11769252378624	
$\sum((A_4 \times G_2(4)) : 2)$	-	2246400	-	

Table IV
The codimensions $d_{nX} = \dim(V/C_V(nX))$

d_{2A}	d_{2B}	d_{2C}	d_{3A}	d_{3B}	d_{3C}	d_{3D}	d_{4A}	d_{4B}	d_{4C}	d_{4D}	d_{4E}
8	12	12	24	12	18	16	16	16	14	16	18
d_{4F}	d_{5A}	d_{5B}	d_{5C}	d_{6A}	d_{6B}	d_{6C}	d_{6D}	d_{6E}	d_{6F}	d_{6G}	d_{6H}
18	24	16	20	24	24	18	18	16	20	18	20
d_{6I}	d_{7A}	d_{7B}	d_{8A}	d_{8B}	d_{8C}	d_{8D}	d_{8E}	d_{8F}	d_{9A}	d_{9B}	d_{9C}
20	24	18	20	20	20	20	18	20	24	22	20
d_{10A}	d_{10B}	d_{10C}	d_{10D}	d_{10E}	d_{10F}	d_{11A}	d_{12A}	d_{12B}	d_{12C}	d_{12D}	d_{12E}
24	24	20	20	20	20	20	24	24	24	22	20
d_{12F}	d_{12G}	d_{12H}	d_{12I}	d_{12J}	d_{12K}	d_{12L}	d_{12M}	d_{13A}	d_{14A}	d_{14B}	d_{15A}
24	20	20	20	20	22	22	22	24	24	20	24
d_{15B}	d_{15C}	d_{15D}	d_{15E}	d_{16A}	d_{16B}	d_{18A}	d_{18B}	d_{18C}	d_{20A}	d_{20B}	d_{20C}
24	24	20	22	22	22	24	22	22	24	22	22
d_{21A}	d_{21B}	d_{21C}	d_{22A}	d_{23A}	d_{23B}	d_{24A}	d_{24B}	d_{24C}	d_{24D}	d_{24E}	d_{24F}
24	24	22	22	22	22	24	24	22	24	22	22
d_{26A}	d_{28A}	d_{28B}	d_{30A}	d_{30B}	d_{30C}	d_{30D}	d_{30E}	d_{33A}	d_{35A}	d_{36A}	d_{39A}
24	22	24	24	24	24	22	22	24	24	24	24
d_{39B}	d_{40A}	d_{42A}	d_{42B}								
24	24	24	24	-	-	-	-	-	-	-	-

Table V
Calculations of $\Sigma(3.Suz.2)$ and $\Sigma((A_4 \times G_2(4)) : 2)$
for some Conjugacy Classes

$(pX, 6B, 39A)$	$2A$	$2B$	$2C$	$3B$
$\Delta(Co_1)$	39	3250	21099	126919
$\Sigma(3.Suz.2)$	0	442	3003	52
$\Sigma((A_4 \times G_2(4)) : 2)$	0	208	39	39
$(pX, 6B, 39A)$	$3C$	$3D$	$5A$	$5B$
$\Delta(Co_1)$	684606	15918279	2367313	235325649
$\Sigma(3.Suz.2)$	78	1716	1300	6240
$\Sigma((A_4 \times G_2(4)) : 2)$	-	22503	416	416

Table V (Continued)

$(pX, 6B, 39A)$	$5C$	$7A$	$7B$	$11A$
$\Delta(Co_1)$	568186164	467467806	7280399841	130072261293
$\Sigma(3.Suz.2)$	-	25506	-	168870
$\Sigma((A_4 \times G_2(4)) : 2)$	-	3120	-	-

$(pX, 6B, 39A)$	$13A$	$23A$		
$\Delta(Co_1)$	54902452176	373637514861		
$\Sigma(3.Suz.2)$	299208	-		
$\Sigma((A_4 \times G_2(4)) : 2)$	508209	-		

$(pX, 6C, 39A)$	$2A$	$2B$	$2C$	$3B$
$\Delta(Co_1)$	130	13364	67782	411320
$\Sigma(3.Suz.2)$	52	1508	0	6734

$(pX, 6C, 39A)$	$3C$	$3D$	$5A$	$5B$
$\Delta(Co_1)$	2196272	48529728	9937200	723057088
$\Sigma(3.Suz.2)$	12272	213408	140868	774592

$(pX, 6C, 39A)$	$5C$	$7A$	$7B$	$11A$
$\Delta(Co_1)$	1797429504	1577601792	22508733312	404140778496
$\Sigma(3.Suz.2)$	-	2882880	-	20953920

$(pX, 6C, 39A)$	$13A$	$23A$		
$\Delta(Co_1)$	172186490112	1162368589824		
$\Sigma(3.Suz.2)$	36040992	-		

$(pX, 6D, 39A)$	$2A$	$2B$	$2C$	$3B$
$\Delta(Co_1)$	364	11804	46956	374088
$\Sigma(3.Suz.2)$	52	884	0	5304

$(pX, 6D, 39A)$	$3C$	$3D$	$5A$	$5B$
$\Delta(Co_1)$	2114736	51115584	6617520	733280704
$\Sigma(3.Suz.2)$	13728	213408	88452	722176

$(pX, 6D, 39A)$	$5C$	$7A$	$7B$	$11A$
$\Delta(Co_1)$	1741988352	1401663744	22583134080	404676679680
$\Sigma(3.Suz.2)$	-	2321280	-	20953920

$(pX, 6D, 39A)$	$13A$	$23A$		
$\Delta(Co_1)$	170003228928	26978640		
$\Sigma(3.Suz.2)$	33644832	-		

$(pX, 6E, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	1131	90090	486681	91
$\Sigma(3.Suz.2)$	117	2964	0	52
$\Sigma((A_4 \times G_2(4)) : 2)$	13	351	936	13

$(pX, 6E, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	3011164	16152760	369686304	64394616
$\Sigma(3.Suz.2)$	13078	23842	400608	267579
$\Sigma((A_4 \times G_2(4)) : 2)$	52	-	34840	3736

$(pX, 6E, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	5463820128	13344978816	11327112576	169344527040
$\Sigma(3.Suz.2)$	1467492	-	5456880	-
$\Sigma((A_4 \times G_2(4)) : 2)$	8736	-	61984	-

$(pX, 6E, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	3033471168000	1285433285760	8717997330432	
$\Sigma(3.Suz.2)$	39323700	67869360	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	0	-	

Table V (Continued)

$(pX, 6F, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	3276	133536	678444	156
$\Sigma(3.Suz.2)$	234	5694	0	78
$(pX, 6F, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	4558320	21199776	492336000	86305440
$\Sigma(3.Suz.2)$	26286	51480	854256	522522
$(pX, 6F, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	7446681216	17791687680	15048224256	227550286080
$\Sigma(3.Suz.2)$	3074448	-	11132160	-
$(pX, 6F, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	4051698094080	1712375827968	11623685898240	
$\Sigma(3.Suz.2)$	83881200	142612080	-	
$(pX, 6G, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	11310	577356	3066882	702
$\Sigma(3.Suz.2)$	0	9789	63609	0
$(pX, 6G, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	19796088	95949360	2211623232	392673840
$\Sigma(3.Suz.2)$	0	0	0	0
$(pX, 6G, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	33260799936	80159249664	68054029056	1021444594560
$\Sigma(3.Suz.2)$	0	-	0	-
$(pX, 6G, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	18221961139200	7711209043200	52307207626752	
$\Sigma(3.Suz.2)$	0	0	-	
$(pX, 6H, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	30888	1436136	7572552	3133
$\Sigma(3.Suz.2)$	1872	19340	98553	673
$\Sigma((A_4 \times G_2(4)) : 2)$	312	9321	15912	312
$(pX, 6H, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	49853011	228198646	5295513171	984104693
$\Sigma(3.Suz.2)$	179179	355810	5766618	3458975
$\Sigma((A_4 \times G_2(4)) : 2)$	1599	-	710619	139776
$(pX, 6H, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	80214260581	192816555204	164884584566	2455222333941
$\Sigma(3.Suz.2)$	207571100	-	74131655	-
$\Sigma((A_4 \times G_2(4)) : 2)$	139776	-	998400	-
$(pX, 6H, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	43747652095537	18521670619920	125537649700593	
$\Sigma(3.Suz.2)$	566095075	957999900	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	5987475	-	
$(pX, 6I, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	80964	3560856	18483348	4056
$\Sigma(3.Suz.2)$	0	15483	98553	0
$\Sigma((A_4 \times G_2(4)) : 2)$	0	0	0	0

Table V (Continued)

$(pX, 6I, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	121910880	574946112	13259980032	2351454144
$\Sigma(3.Suz.2)$	0	0	0	0
$\Sigma((A_4 \times G_2(4)) : 2)$	936	-	344448	0

$(pX, 6I, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	200477292288	400952442880	407918484480	6138513338880
$\Sigma(3.Suz.2)$	0	-	0	-
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	-

$(pX, 6I, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	109369130557440	46259137207296	313840450879488	
$\Sigma(3.Suz.2)$	0	0	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	0	-	

$(pX, 8A, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	442	23816	105846	26
$\Sigma(3.Suz.2)$	208	4472	4338	0
$\Sigma((A_4 \times G_2(4)) : 2)$	0	390	0	0

$(pX, 8A, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	69784	3655600	83063552	15185040
$\Sigma(3.Suz.2)$	19942	450760	720512	353184
$\Sigma((A_4 \times G_2(4)) : 2)$	104	-	42432	0

$(pX, 8A, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	1235351936	3008668416	2580554496	38138695296
$\Sigma(3.Suz.2)$	2502656	-	8442720	-
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	-

$(pX, 8A, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	682702072832	289526212352	1961478245376	
$\Sigma(3.Suz.2)$	70803200	10491520	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	1138176	-	

$(pX, 8B, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	7488	314652	1673100	156
$\Sigma(3.Suz.3)$	0	11232	72072	0
$\Sigma((A_4 \times G_2(4)) : 2)$	0	2106	0	0

$(pX, 8B, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	11021712	54261168	1246147968	205735920
$\Sigma(3.Suz.3)$	0	0	0	0
$\Sigma((A_4 \times G_2(4)) : 2)$	312	-	127296	0

$(pX, 8B, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	18740147712	44943515136	37635217152	575076528000
$\Sigma(3.Suz.3)$	0	-	0	-
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	-

$(pX, 8B, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	10251161456640	4330802521344	29422556587008	
$\Sigma(3.Suz.2)$	0	0	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	0	3324672	-	

Table V (Continued)

$(pX, 8C, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	6812	314860	1575288	208
$\Sigma(3.Suz.2)$	650	11752	0	91
$\Sigma((A_4 \times G_2(4)) : 2)$	0	1092	0	0

$(pX, 8C, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	10542896	54194816	12551119424	203970000
$\Sigma(3.Suz.2)$	61256	139568	2162368	1062048
$\Sigma((A_4 \times G_2(4)) : 2)$	156	-	44928	0

$(pX, 8C, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	18634208320	44836311936	37480235520	573582472320
$\Sigma(3.Suz.2)$	7552480	-	25272000	-
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	-

$(pX, 8C, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	10246558466560	43285439324208	29422748040192	
$\Sigma(3.Suz.2)$	212243200	348404992	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	2246400	-	

$(pX, 8D, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	25818	1024608	5102838	1170
$\Sigma((A_4 \times G_2(4)) : 2)$	0	0	0	0

$(pX, 8D, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	34673808	161133024	3738596160	655292976
$\Sigma((A_4 \times G_2(4)) : 2)$	468	-	134784	0

$(pX, 8D, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	56557041216	135121963392	114290801664	1727964950400
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	-

$(pX, 8D, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	30767288547840	13003788824832	88267229568432	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	0	-	

$(pX, 8F, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	55120	2659904	13932048	4472
$\Sigma((A_4 \times G_2(4)) : 2)$	0	3588	0	0

$(pX, 8F, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	90953824	430812928	9938150144	1805425024
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	0

$(pX, 8F, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	149944367872	361109349072	307736326144	4598897372160
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	-

$(pX, 8F, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	82007658803200	34715251410944	235382073778176	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	6709248	-	

$(pX, 9A, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	29016	1085682	5228184	676
$\Sigma(3.Suz.2)$	1560	29640	0	286

$(pX, 9A, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	36114832	168672244	39598781240	659452404
$\Sigma(3.Suz.2)$	149032	326872	5125536	2655900

Table V (Continued)

$(pX, 9A, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	59668847784	141940172556	119021477796	1821187031040
$\Sigma(3.Suz.2)$	18031104	-	61401600	-
$(pX, 9A, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	32416738099200	13684013431452	9299033225116	
$\Sigma(3.Suz.2)$	503193600	832145184	-	
$(pX, 9B, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	98007	4263051	21610680	5969
$\Sigma(3.Suz.2)$	1716	39000	0	780
$(pX, 9B, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	144466296	678185664	15754201359	2790424338
$\Sigma(3.Suz.2)$	169650	305760	5125536	3484260
$(pX, 9B, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	237641149746	569697022440	483065223615	7274051665920
$\Sigma(3.Suz.2)$	18869760	-	70387200	-
$(pX, 9B, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	129622671360000	54817446264282	371961222866064	
$\Sigma(3.Suz.2)$	503305920	871097760	-	
$(pX, 9C, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	81705	4241523	222590160	7605
$\Sigma(3.Suz.2)$	1716	34320	0	624
$(pX, 9C, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	143917176	682165068	15669063657	2927143674
$\Sigma(3.Suz.2)$	159198	316368	5125536	3085524
$(pX, 9C, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	36608603686	571627251468	489921046173	7263355207680
$\Sigma(3.Suz.2)$	18450432	-	65894400	-
$(pX, 9C, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	129575974993920	54900973913994	371961282571164	
$\Sigma(3.Suz.2)$	503081280	851419296	-	
$(pX, 10A, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	5707	226538	1040715	195
$\Sigma(3.Suz.2)$	1027	18824	0	117
$\Sigma((A_4 \times G_2(4)):2)$	0	0	0	0
$(pX, 10A, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	7225400	34258640	801270080	138343504
$\Sigma(3.Suz.2)$	98033	223262	3459560	1694017
$\Sigma((A_4 \times G_2(4)):2)$	416	-	139776	0
$(pX, 10A, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	12035897536	28792777728	24341175040	368128166016
$\Sigma(3.Suz.2)$	12083968	-	40436032	-
$\Sigma((A_4 \times G_2(4)):2)$	0	-	0	-
$(pX, 10A, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	6561889804288	2773855315712	301288174387200	
$\Sigma(3.Suz.2)$	339639664	557521640	-	
$\Sigma((A_4 \times G_2(4)):2)$	-	0	-	

Table V (Continued)

$(pX, 10B, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	40872	1693536	8672040	1170
$\Sigma(3.Suz.2)$	0	22347	141687	0
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	0
$(pX, 10B, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	57763446	275753868	6383438646	1083394650
$\Sigma(3.Suz.2)$	0	0	0	0
$\Sigma((A_4 \times G_2(4)) : 2)$	1326	-	447174	0
$(pX, 10B, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	96242422458	230341188168	193854157068	2946776427738
$\Sigma(3.Suz.2)$	0	-	0	-
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	-
$(pX, 10B, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	52497118922898	22183972860192	150644112698898	
$\Sigma(3.Suz.2)$	0	0	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	0	-	
$(pX, 12A, 39A)$	$2A$	$2B$	$2C$	$3B$
$\Delta(Co_1)$	247	15834	77571	500344
$\Sigma(3.Suz.2)$	130	2886	0	14209
$\Sigma((A_4 \times G_2(4)) : 2)$	0	39	117	52
$(pX, 12A, 39A)$	$3C$	$3D$	$5A$	$5B$
$\Delta(Co_1)$	2419872	54961088	10964408	828651616
$\Sigma(3.Suz.2)$	29952	486512	269594	1719016
$\Sigma((A_4 \times G_2(4)) : 2)$	-	32032	1040	1040
$(pX, 12A, 39A)$	$5C$	$7A$	$7B$	$11A$
$\Delta(Co_1)$	2015637312	1748431360	25495763904	455425674496
$\Sigma(3.Suz.2)$	-	6007924	-	47739328
$\Sigma((A_4 \times G_2(4)) : 2)$	-	7592	-	-
$(pX, 12A, 39A)$	$13A$	$23A$		
$\Delta(Co_1)$	193261547648	1307666899968		
$\Sigma(3.Suz.2)$	79717976	-		
$\Sigma((A_4 \times G_2(4)) : 2)$		-		
$(pX, 12B, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	5798	224172	1123590	130
$\Sigma(3.Suz.2)$	26	624	0	13
$\Sigma((A_4 \times G_2(4)) : 2)$	0	0	0	0
$(pX, 12B, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	7681752	35396608	834518464	139251112
$\Sigma(3.Suz.2)$	2847	5356	90064	59254
$\Sigma((A_4 \times G_2(4)) : 2)$	624	-	177216	0
$(pX, 12B, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	12588032288	2996393328	25133049344	384240816960
$\Sigma(3.Suz.2)$	329576	-	1213160	-
$\Sigma((A_4 \times G_2(4)) : 2)$	0	-	0	-

Table V (Continued)

$(pX, 12B, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	6837905135360	2887102950784	19615446948864	
$\Sigma(3.Suz.2)$	8832200	15222376	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	0	-	
$(pX, 12C, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	9373	455598	2310009	637
$\Sigma(3.Suz.2)$	910	26169	48594	208
$\Sigma((A_4 \times G_2(4)) : 2)$	0	2587	273	0
$(pX, 12C, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	15157376	71537440	1658908160	301895152
$\Sigma(3.Suz.2)$	90701	192166	3063008	1670890
$\Sigma((A_4 \times G_2(4)) : 2)$	364	-	151528	2080
$(pX, 12C, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	25020941504	60187380864	51339525120	766693249920
$\Sigma(3.Suz.2)$	10849592	-	37615500	-
$\Sigma((A_4 \times G_2(4)) : 2)$	2080	-	14768	-
$(pX, 12C, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	13669288655360	5786170707200	39230405720064	
$\Sigma(3.Suz.2)$	300731600	501290192	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	4462848	-	
$(pX, 15A, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	7930	383500	2057328	494
$\Sigma(3.Suz.2)$	52	1300	0	26
$\Sigma((A_4 \times G_2(4)) : 2)$	286	5434	13728	286
$(pX, 15A, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	13216840	61217780	1407652922	261442376
$\Sigma(3.Suz.2)$	5356	8840	153686	116090
$\Sigma((A_4 \times G_2(4)) : 2)$	1144	-	488670	117676
$(pX, 15A, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	21382926188	51473010732	44082174318	654815879640
$\Sigma(3.Suz.2)$	579332	-	2246868	-
$\Sigma((A_4 \times G_2(4)) : 2)$	117676	-	838422	-
$(pX, 15A, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	11665691061752	4941297821096	33476433671772	
$\Sigma(3.Suz.2)$	15108704	26735696	0	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	4462848	-	
$(pX, 15C, 39A)$	$2A$	$2B$	$2C$	$3A$
$\Delta(Co_1)$	251667	11533197	59979816	21957
$\Sigma(3.Suz.2)$	6006	123630	0	2028
$\Sigma((A_4 \times G_2(4)) : 2)$	286	13234	13728	286
$(pX, 15C, 39A)$	$3B$	$3C$	$3D$	$5A$
$\Delta(Co_1)$	395868642	1832694864	42379054029	77999250068
$\Sigma(3.Suz.2)$	573456	1138800	18450822	11072360
$\Sigma((A_4 \times G_2(4)) : 2)$	1144	-	458670	117676
$(pX, 15C, 39A)$	$5B$	$5C$	$7A$	$7B$
$\Delta(Co_1)$	641602913224	1541658397188	1316013938603	19641096313878
$\Sigma(3.Suz.2)$	66422356	-	237216980	-
$\Sigma((A_4 \times G_2(4)) : 2)$	117676	-	838422	-

Table V (Continued)

$(pX, 15C, 39A)$	$11A$	$13A$	$23A$	
$\Delta(Co_1)$	349981218038686	148137357811638	1004293857624330	
$\Sigma(3.Suz.2)$	1811489680	3065623392	-	
$\Sigma((A_4 \times G_2(4)) : 2)$	-	13478400	-	

Table VI
Partial Fusion Maps of $3.Suz.2$ and $(A_4 \times G_2(4)) : 2)$ into Co_1

$3 \cdot Suz \cdot 2$-classes	2a	2b	2c	2d	3a	3b	3c	3d	3e
$\rightarrow Co_1$	2A	2B	2B	2C	3A	3A	3B	3B	3C
$3 \cdot Suz \cdot 2$-classes	3f	4a	4b	4c	4d	4e	4f	5a	5b
$\rightarrow Co_1$	3D	4A	4B	4D	4E	4D	4E	5A	5B
$3 \cdot Suz \cdot 2$-classes	6a	6b	6c	6d	6e	6f	6g	6h	6i
$\rightarrow Co_1$	6A	6B	6A	6E	6C	6D	6F	6E	6F
$3 \cdot Suz \cdot 2$-classes	6j	6k	6l	6m	6n	7a	8a	8b	8c
$\rightarrow Co_1$	6H	6B	6G	6H	6I	7A	8A	8C	8F
h						35	112	16	8
$3 \cdot Suz \cdot 2$-classes	8d	8e	8f	8g	8h	9a	9b	9c	10a
$\rightarrow Co_1$	8A	8A	8B	8B	8F	9B	9C	9A	10A
h		112		32	8	3	3	12	40
$3 \cdot Suz \cdot 2$-classes	10b	10c	10d	10e	11a	12a	12b	12c	12d
$\rightarrow Co_1$	10C	10C	10B	10F	11A	12A	12B	12C	12F
h		12	12	4	1	81	1	33	1
$3 \cdot Suz \cdot 2$-classes	12e	12f	12g	12h	12i	12j	12k	12l	12m
$\rightarrow Co_1$	12A	12E	12E	12D	12C	12J	12L	12G	12K
$3 \cdot Suz \cdot 2$-classes	12n	12o	12p	13a	14a	14b	15a	15b	15c
$\rightarrow Co_1$	12C	12J	12L	13A	14A	14A	15A	15B	15C
$3 \cdot Suz \cdot 2$-classes	15d	15e	16a	18a	18b	18c	20a	21a	21b
$\rightarrow Co_1$	15B	15D	16A	18B	18C	18A	20A	21A	21A
$3 \cdot Suz \cdot 2$-classes	21c	21d	22a	24a	24b	24c	24d	24e	24f
$\rightarrow Co_1$	21B	21B	22A	24A	24B	24D	24A	24C	24A
$3 \cdot Suz \cdot 2$-classes	24g	24h	24i	24j	28a	30a	30b	30c	33a
$\rightarrow Co_1$	24A	24C	24E	24D	28B	30A	30B	30B	33A
$3 \cdot Suz \cdot 2$-classes	39a	39b	40a	40b	42a	60a			
$\rightarrow Co_1$	39A	39B	40A	40A	42A	60A			
h	1	1		2	1	1			
$(A_4 \times G_2(4)) : 2$-classes	2a	2b	2c	2d	2e	2f	3a	3b	3c
$\rightarrow Co_1$	2A	2B	2B	2B	2B	2C	3A	3D	3A
$(A_4 \times G_2(4)) : 2$-classes	3d	3e	4a	4b	4c	4d	4e	4f	4g
$\rightarrow Co_1$	3B	3D	4A	4D	4B	4D	4E	4D	4B
$(A_4 \times G_2(4)) : 2$-classes	4h	4i	4j	4k	5a	5b	6a	6b	6c
$\rightarrow Co_1$	4D	4E	4E	4F	5B	5A	6A	6H	6B

Table VI (Continued)

$(A_4 \times G_2(4))$: 2-classes	6d	6e	6f	6g	6h	6i	6j	6k	6l
$\to Co_1$	6H	6B	6H	6B	6I	6A	6B	6E	6H
$(A_4 \times G_2(4))$: 2-classes	7a	8a	8b	8c	8d	8e	8f	8g	8h
$\to Co_1$	7A	8A	8C	8A	8F	8B	8B	8D	8F
$(A_4 \times G_2(4))$: 2-classes	8i	8j	10a	10b	10c	10d	10e	10f	11a
$\to Co_1$	8C	8F	10A	10C	10C	10B	10B	10F	11A
$(A_4 \times G_2(4))$: 2-classes	12a	12b	12c	12d	12e	12f	12g	12h	12i
$\to Co_1$	12A	12C	12C	12C	12L	12C	12B	12B	12A
$(A_4 \times G_2(4))$: 2-classes	12j	12k	12l	12m	12n	12o	12p	12q	12r
$\to Co_1$	12C	12B	12E	12J	12J	12F	12L	12F	12M
$(A_4 \times G_2(4))$: 2-classes	13a	14a	14b	15a	15b	15c	15d	15e	15f
$\to Co_1$	13A	14A	14A	15B	15C	15B	15B	15A	15A
$(A_4 \times G_2(4))$: 2-classes	15g	15h	15i	15j	16a	16b	16c	16d	21a
$\to Co_1$	15D	15D	15C	15C	16A	16A	16B	16B	21A
$(A_4 \times G_2(4))$: 2-classes	21b	21c	21d	24a	24b	24c	24d	24e	24f
$\to Co_1$	21A	21B	21B	24A	24D	24D	24A	24B	24D
$(A_4 \times G_2(4))$: 2-classes	24g	24h	26a	28a	30a	30b	30c	30d	30e
$\to Co_1$	24B	24B	26A	28B	30B	30C	30A	30A	30B
h					3	1	4	4	3
$(A_4 \times G_2(4))$: 2-classes	30f	39a	39b	42a					
$\to Co_1$	30B	39A	39B	42A					
h	3	1	1	1					

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