ACTA MATHEMATICA VIETNAMICA
Volume 28, Number 3, 2003, pp. 309-318

ON VNR RINGS AND P-INJECTIVITY

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Abstract. This note contains the following results: (1) A is strongly regular iff every non-zero factor ring of A is a semi-prime ring containing a non-zero reduced p-injective left ideal which is a left annihilator; (2) A is an ELT von Neumann regular ring iff A is a semi-prime MELT ring whose essential right ideals are idempotent iff A is a semi-prime ELT ring such that for any essential left ideal L of A, either AA/L is p-injective or A/LA is flat; (3) If A is a semi-prime ring whose simple left modules are either YJ-injective or projective, then the Jacobson radical of A is zero. If, further, each maximal right ideal of A is either injective or a two-sided ideal of A, then A is either strongly regular or right self-injective regular. Several conditions are given for a left Noetherian ring to be left Artinian.

1. Introduction

Throughout, A denotes an associative ring with identity and A-modules are unital. J, Z, Y will stand respectively for the Jacobson radical, the left singular ideal and the right singular ideal of A. A is called semi-primitive (resp. (1) left non-singular; (2) right non-singular) if $J = 0$ (resp. (1) $Z = 0$; (2) $Y = 0$). An ideal of A will always mean a two-sided ideal of A. Following S. H. Brown, A is called left (resp. right) quasi-duo if every maximal left (resp. right) ideal of A is an ideal of A (S. H. Brown (1973)). A left (right) ideal of A is called reduced if it contains no non-zero nilpotent element.

In 1974, we introduced p-injective modules to study von Neumann regular ring, V-rings and their generalizations [22]. Following [6], we shall write "A is VNR" whenever A is a von Neumann regular ring. It is well-known that A is VNR iff every left (right) A-module is flat (M. Harada (1956); M. Auslander (1957)). This remains true if "flat" is replaced by "p-injective" [22] or "YJ-injective" [37].

Recall that a right A-module M is (1) p-injective if, for every principal right ideal P of A, any right A-homomorphism of P into M extends to one of A into M; (2) YJ-injective if, for every $0 \neq b \in A$, there exist a positive integer n such that $b^n \neq 0$ and any right A-homomorphism of b^nA into M extends to one of A into M ([20], [30], [32], [35]).

Received August 7, 2002; in revised form September 1, 2003.
1991 *Mathematics Subject Classification.* 16D40; 16D50; 16E50; 16N60.
Key words and phrases. von Neumann regular; p-injectivity; YJ-injectivity; idempotent ideal; strongly regular.

A is called right p-injective (resp. right YJ-injective) if A_A is p-injective (resp. YJ-injective). Similarly, p-injectivity or YJ-injectivity is defined on the left side. If A is right YJ-injective, then the Jacobson radical J of A coincides with the right singular ideal Y of A [30, Proposition 1] (this extends the well-known result for right self-injective rings). Also, A is right YJ-injective iff for any $0 \neq b \in A$, there exist a positive integer n such that Ab^n is a non-zero left annihilator [32, Lemma 3]. YJ-injectivity is also called GP-injectivity in the literature ([11], [13]).

K. R. Goodearl's textbook [8] has motivated a large number of papers on VNR rings during the last twenty years. Following the study of flat modules over non-VNR rings, various authors have considered p-injective and YJ-injective modules over rings which are not necessarily VNR (cf. for example, [2], [3], [11], [13], [20], [36], [37]). As usual, A is called fully idempotent (resp. (a) fully left idempotent; (b) fully right idempotent) if every ideal (resp. (a) left ideal; (b) right ideal) of A is idempotent.

The study of strongly regular rings was initiated by R. F. Arens - I. Kaplansky (1948).

We start with a new characterization of strongly regular rings.

Theorem 1. *The following conditions are equivalent:*

(1) *A is strongly regular;*

(2) *Every non-zero factor ring of A is a semi-prime ring containing a non-zero reduced p-injective left ideal which is a left annihilator;*

(3) *Every non-zero factor ring of A is a semi-prime ring containing a non-zero reduced YJ-injective left ideal which is a left annihilator.*

Proof. Since every factor ring of a strongly regular ring is strongly regular, then (1) implies (2).

(2) implies (3) evidently.

Assume (3). Let B be a non-zero prime factor ring of A. Then B contains a non-zero reduced YJ-injective left ideal K which is a left annihilator. By [26, Proposition 6], B is an integral domain. Let $0 \neq k \in K$. There exist a positive integer n such that any left B-homomorphism of Bk^n into K extends to one of B into K. If $i : Bk^n \to K$ is the natural inclusion, there exist $s \in K$ such that $k^n = i(k^n) = k^n s$ which implies $s = 1 \in K$, whence $K = B$. Since ${}_BB$ is YJ-injective, for any $0 \neq c \in B$, there exist a positive integer m such that if $g : Bc^m \to B$ is the left B-homomorphism defined by $g(bc^m) = b$ for all $b \in B$, there exist $d \in B$ such that $1 = g(c^m) = c^m d$ which proves that B is a division ring. Since A is a fully idempotent ring, then A is VNR by [8, Corollary 1.18]. Therefore (3) implies (1) by [8, Theorem 3.2]. □

A special case of left bounded rings [5, p.49] is the class of ELT rings. Recall that A is ELT if every essential left ideal of A is an ideal of A. We also say that A is MELT if any maximal essential left ideal (if it exists) is an ideal of A. ERT and MERT rings are similarly defined on the right side.

Remark 1. If A is VNR, then the above four terms are equivalent properties for the ring A (cf. [33, p.56]).

In [28, Question 2], it is asked whether a prime MELT left or right self-injective ring A is Artinian? We know that the answer is "yes" if A has non-zero socle. Consequently, this question may be reformulated as follows:

Question 1. Does there exist a prime left or right self-injective ring which is left quasi-duo but not a division ring?

J. S. Alin-E. P. Armendariz [1] initiated the study of rings whose simple modules are either injective or projective (later called generalized V-rings by V. S. Ramamurthy-K. M. Rangaswamy [16]) (cf. also [2]). We consider rings whose simple right modules are either YJ-injective or projective. Note that in a semi-prime ring A, if L is an essential left ideal which is an ideal of A, then L is an essential right ideal of A.

The next proposition improves [25, Proposition 9(4)] and [29, Proposition 2(2)].

Proposition 1. *The following conditions are equivalent:*

(1) *A is an ELT VNR ring;*

(2) *A is a semi-prime MELT ring whose simple right modules are either injective or projective;*

(3) *A is a semi-prime MELT ring whose simple right modules are either p-injective or projective;*

(4) *A is a semi-prime MELT ring whose essential right ideals are idempotent;*

(5) *A is is a semi-prime ELT ring such that, for any essential left ideal L of A, either ${}_AA/L$ is p-injective or A/L_A is flat.*

Proof. Assume (1). Then A is a semi-prime MELT ring which is also ERT [33, p.56]. For any maximal right ideal R of A, if A/R_A is not projective, then R is an essential right ideal which is therefore an ideal of A. Since ${}_AA/R$ is flat, then by [29, Lemma 1], A/R_A is injective. Thus (1) implies (2).

(2) implies (3) evidently.

(3) implies (4) by [24, Proposition 6].

Assume (4). Since A is MELT and every essential right ideal of A is idempotent, then any factor ring of A has the same two properties. Let B be a prime factor ring of A. Then B is MELT and every essential right ideal of B is idempotent. For any $0 \neq t \in B$, $T = BtB$ is a non-zero ideal of B which is therefore an essential right ideal of B. There exist a complement right subideal K of T such that $R = tB \oplus K$ is an essential right subideal of T. Therefore R is an essential right ideal of B, which is then idempotent. Now $t \in R^2 = R$ implies that

$$t = \sum (tb_i + k_i)(tc_i + s_i),$$

where b_i, $c_i \in B$ and k_i, $s_i \in K$, whence

$$t - \sum tb_i(tc_i + s_i) = \sum k_i(tc_i + s_i) \in B \cap K = 0.$$

Since T is an ideal of B, then

$$t = \sum tb_i(tc_i + s_i) \in tT = (tB)^2,$$

which proves that $tB = (tB)^2$. We have just proved that B is a fully right idempotent ring. Since B is MELT, by [34, Proposition 9], B is VNR. For any ideal I of A, let C be a complement right ideal of A such that $E = I \oplus C$ is an essential right ideal of A. Then $E = E^2$. We now have $I \subseteq I(I \oplus C)$. Since $(IC)^2 = I(CI)C = 0$, then $IC = 0$ (in as much as A is semi-prime). This yields $I \subseteq I^2$, whence $I = I^2$. A is therefore fully idempotent and (4) implies (1) by [8, Corollary 1.18].

It is clear that (1) implies (5).

Assume (5). For any $b \in A$, if $I = AbA + l(b)$, K a complement left ideal of A such that $L = I \oplus K$ is an essential left ideal of A, since A is semi-prime, $l(AbA) = r(AbA)$ and therefore

$$AbAK \subseteq AbA \cap K \subseteq I \cap K = 0$$

which implies that

$$K \subseteq r(AbA) = l(AbA) \subseteq l(b),$$

whence $K \subseteq I \cap K = 0$. This proves that $I = L$ is an essential left ideal of A. Therefore I is an ideal of A. First suppose that ${}_AA/I$ is p-injective. Define a left A-homomorphism

$$f : Ab \to A/I \text{ by } f(ab) = a + I \text{ for all } a \in A.$$

There exist $y \in A$ such that $1 + I = f(b) = by + I$, which yields $1 - by = c + u$, $c \in AbA$, $u \in l(b)$. Now

$$b = byb + cb + ub = byb + cb(Ab)^2$$

which proves that $Ab = (Ab)^2$. Next suppose that A/I_A is flat. Since $b \in I$, we have $b \in Ib$ [4, p.458]. If $b = wb$, $w \in I$, let $w = s + t$, where $s \in AbA$, $t \in l(b)$. Then $b = sb + tb = sb \in (Ab)^2$ again and we have $Ab = (Ab)^2$. We have proved that in any case A is a fully left idempotent ring. By [2, Theorem 3.1], A is VNR and hence (5) implies (1). □

If every singular right A-module is injective, then A is a right hereditary ring (K. R. Goodearl, Singular torsion and the splitting properties, Amer. Math. Soc. Memoirs, Vol.124 (1972)). Such rings are noted right SI-rings. (Goodearl's result remains valid if "singular right A-module" is replaced by "divisible singular right A-module" [27, p.192]).

In connection with this type of result, [21, Theorem 4] asserts that if A is right non-singular, then the singular submodule of any injective right A-module is injective (cf. Abraham ZAK's remark in Math. Reviews 40 (1970)#5664 and also [5, p.88]).

We recall the following theorem of K. Goodearl in the above memoir (cf. [2, Theorem 2.7]).

Theorem 2. (K. Goodearl) *A is a right SI-ring if, and only if, A decomposes as:*

$$A = S \times A_1 \times \dots \ \cdots \times A_n,$$

where $S_{oc}.S_s$ is essential in S_s and each A_i is a simple right SI-ring which is Morita equivalent to a domain.

Remark 2. Proposition 1 and Goodearl's theorem imply that if A is a semi-prime MELT right SI-ring, then $A = S \times A_1 \times dots \ \cdots \times A_n$, where S is an ERT VNR right hereditary ring with essential right socle and each A_i is a simple Artinian ring.

We know that A is VNR if every singular right A-module is flat [25, Theorem 5]. As usual, A is called a right IF-ring if every injective right A-module is flat.

Remark 3. If A is a right IF-ring whose singular right modules are injective, then $A = S \times A_1 \times \dots \ \cdots \times A_n$, where S is a VNR right hereditary ring with essential right socle and each A_i is a simple VNR right hereditary ring.

In Remark 2, the term "semi-prime" cannot be omitted. Indeed, ELT rings whose singular right modules are injective need not be VNR, as shown by the following example.

Example. Let K be a field and

$$A = \begin{pmatrix} K & K \\ 0 & k \end{pmatrix}.$$

A has only one proper essential left ideal

$$L = \begin{pmatrix} K & K \\ 0 & 0 \end{pmatrix}$$

and L is an ideal of A. Therefore A is ELT. All singular right (and left) A-modules are injective. Although A is left and right hereditary, Artinian, A is not VNR. But A is left and right quasi-duo.

The Jacobson radical J, the right and left singular ideals Y, Z of A respectively are fundamental concepts in ring theory (cf. [5], [6], [7], [8], [19]).

Proposition 2. *Suppose that every simple right A-module is either YJ-injective or projective. Then $Y \cap J = 0$.*

Proof. Suppose that $Y \cap J$ is a non-zero reduced ideal of A. If $0 \neq w \in Y \cap J$, $r(w)$ is an essential right ideal of A and $wA \cap r(w) \neq 0$. Let $b \in A$ such that $0 \neq wb \in r(w)$. Since $Y \cap J$ is reduced, $wb \in Y \cap J$, $wbw \in Y \cap J$, then

$$(wbw)^2 = wb(w^2b)w = 0$$

implies that $wbw = 0$ and then

$$(wb)^2 = (wbw)b = 0$$

implies that $wb = 0$, a contradiction! This proves that if $Y \cap J \neq 0$, then $Y \cap J$ contains a non-zero nilpotent element. There exist $0 \neq z \in Y \cap J$ such that

$z^2 = 0$. Now $L = AzA + r(z)$ is an essential right ideal of A. If we suppose that $L \neq A$, let M be a maximal right ideal of A containing L. Then A/M_A is simple non-projective (because M_A is essential in A_A) which implies that A/M_A is YJ-injective. Since $z \neq 0$, $z^2 = 0$, define a right A-homomorphism $g : zA \to A/M$ by $g(za) = a + M$ for all $a \in A$. Then there exist $d \in A$ such that $g(z) = dz + M$ which yields $1 - dz \in M$. Since $dz \in L \subseteq M$, then $1 \in M$ which contradicts $M \neq A$. This proves that $L = A$. Now $1 = c + u$, $c \in AzA$, $u \in r(z)$, and $z = zc + zu = zc$. Since $c \in AzA \subseteq J$, $1 - c$ is right invertible in A which implies that $z = 0$, a contradiction! We finally have $Y \cap J = 0$. □

The next corollary follows from a well-known result of Y. Utumi [18, Lemma 4.1].

Corollary 1. *If A is a right continuous ring whose simple right modules are either YJ-injective or projective, then A is VNR.*

[16, Theorem 3.3] toghether with Proposition 2 yield the following characterization of generalized V-rings.

Corollary 2. *The following conditions are equivalent:*

(1) *Every simple right A-module is either injective or projective;*

(2) *Every simple right A-module is either YJ-injective or projective and every proper essential right ideal of A is the intersection of maximal right ideals of A.*

If A is left non-singular, it is well-known that the injective hull of ${}_AA$ is a left self-injective regular ring. For an arbitrary ring A, the injective hull needs not be a ring and it is not always possible to embed A in a self-injective ring [6, p.309]. However, P. Menal-P. Vamos [12] showed that any ring may be embedded in FP-injective ring. Consequently, any ring may be embedded in a p-injective ring. This enhances the attention paid to p-injective rings (cf. for example, [6, Theorem 6.4], [9], [14], [15]). Some authors prefer the expression "principally injective" in full (cf. for example T. Y. Lam: Lectures on modules and rings, Graduate texts in Math. Springer (1998)). However, the term "p-injective" is used by Wisbauer [19] and Faith [6].

Remark 4. If A is a left p-injective ring, then any finitely generated projective left a-module is p-injective.

Note that in a semi-prime ring A, the sum of all reduced ideals of A is the unique maximal reduced ideal of A [31, Lemma 1.].

Proposition 3. *Let A be a left p-injective ring such that $A = B \oplus C$, where B, C are ideals of A. Then B and C are left p-injective rings.*

In general, a semi-prime left p-injective ring A needs not be regular (even if A is a P.I. ring) (cf. [3,p.853]).

Corollary 3. *Let A be a semi-prime left p-injective ring such that $A = B \oplus C$, where B, C are ideals of A, B being the sum of all reduced ideals of A, C being* *a left p.p. ring. Then A is the direct sum of a VNR ring and a strongly regular ring.*

Proof. B is a reduced left p-injective ring and therefore strongly regular by [23, Theorem 1]. C is a left p-injective left p. p. ring and therefore VNR. □

We propose a nice result which is quite general.

Proposition 4. *Let A be a semi-prime ring whose simple left modules are either YJ-injective or projective. Then A is semi-primitive.*

Proof. We first prove that J is reduced. Suppose the contrary: Let $0 \neq c \in J$ such that $c^2 = 0$. If $Ac \neq (Ac)^2$, we deduce a contradiction. The set of left ideals I of A such that $(Ac)^2 \subseteq I \subset Ac$ has, by Zorn's Lemma, a maximal member M. Then ${}_AAc/M$ is simple. Now, for any left subideal K of Ac such that $K \cap (Ac)^2 = 0$, we have $K^2 \subseteq K \cap (Ac)^2 = 0$ which implies $K = 0$ (because A is semi-prime). Therefore ${}_A(Ac)^2$ is essential in ${}_AAc$ which implies that ${}_AM$ is essential in ${}_AAc$. By hypothesis, ${}_AAc/M$ is YJ-injective. Define a left A-homomorpjism $g : Ac \to Ac/M$ by $g(ac) = ac + M$ for all $a \in A$. There exist $d \in A$ such that

$$c + M = g(c) = cdc + M.$$

Then $c - cdc \in M$ which yields $c \in M$ (since $cdc \in (Ac)^2 \subseteq M$), whence $M = Ac$, a contradiction! Therefore $Ac = (Ac)^2$ which implies that $c = uc$, where $u \in AcA \subseteq J$. Since $1 - u$ is left invertible in A, $c = 0$ which contradicts our original assumption. This proves that J is reduced.

We now prove that $J = 0$. If not, let $0 \neq v \in J$. Let K be a complement left ideal of A such that $L = (AvA + l(v)) \oplus K$ is an essential left ideal of A. Then $vK \subseteq AvA \cap K = 0$ which implies that $(Kv)^2 = 0$, whence $Kv = 0$. Therefore $K \subseteq l(v)$ which yields $K = K \cap l(v) = 0$, showing that $L = AvA + l(v)$ is an essential left ideal. If $L \neq A$, let N be a maximal left ideal of A such taht $L \subset N$. Then ${}_AA/N$ is simple, YJ-injective. There exist a positive integer m such that any left A-homomorphism of Av^m into A/N extends to A. Since A is reduced, we may define a left A-homomorphism $h : Av^m \to A/N$ by $h(av^m) = a + N$ for all $a \in A$. There exist $w \in A$ such that $h(v^m) = v^m w + N$. Now $1 + N = h(v^m)$ implies that $1 - v^m w \in N$, whence $1 \in N$, contradicting $N \neq A$. This proves that $L = A$. If $1 = s + t$, $s \in AvA$, $t \in l(v)$, then $v = sv + tv = sv$ and since $s \in AvA \subseteq J$, $1 - s$ is left invertible in A which yields $v = 0$, a contradiction. We have proved that $J = 0$. □

Corollary 4. *Let A be a semi-prime ring whose simple left modules are either YJ-injective or projective. If each maximal right ideal of A is either injective or an ideal of A, then A is either strongly regular or right self-injective regular.*

Proof. First suppose that each maximal right ideal of A is an ideal of A. Since A is semi-primitive by proposition 4 and right quasi-duo, then A is a reduced ring (cf. R. Yue Chi Ming, On von Neumann regular rings VI, Rend. Sem. Mat.

Univ. Torino **39** (1981), 75-84 (p. 82)). By [11, Proposition 18], A is strongly regular. Now suppose there exist a maximal right ideal M which is not an ideal of A. Then M_A is injective and by [34, Lemma 4], A is right self-injective. Since J=0, A is VNR. □

In Corollary 4, the term "semi-prime" is not superfluous (cf. the example given above). Another remark on p-injective rings.

Remark 5. If A is a left p-injective ring such that (a) every complement left ideal is a direct summand of ${}_AA$ and (b) every simple left A-module is either YJ-injective or projective, then A is VNR. (Rings satisfying condition (a) are studied in [10]).

We now give various conditions for left Noetherian rings to be left Artinian.

Theorem 3. *The following conditions are equivalent:*

(1) *A is left Artinian;*

(2) *A is a left Noetherian ring such that any non-zero prime factor ring B satisfies any one of the following conditions:* (a) *B has non-zero socle;* (b) *B contains a p-injective maximal left ideal;* (c) *B is left YJ-injective;* (d) *B is right YJ-injective.*

Proof. (1) implies (2) evidently.

Assume (2). Let B be a non-zero prime factor ring of A.

(a) If B has non-zero left (and right) socle S, then ${}_BS$, being essential in ${}_BB$ and also a direct summand of ${}_BB$, implies that $B = S$, which shows that B is simple Artinian.

(b) If B contains a p-injective maximal left ideal K, then ${}_BK$ is finitely generated (since B is left Noetherian) and given ${}_BK$ is p-injective, then B/K is a finitely related flat left B-module which implies that ${}_BB/K$ is projective, whence $B = K \oplus V$, where V is a minimal projective left ideal of B. Since B is prime, K cannot be an ideal of B and the proof of [34, Lemma 4] shows that B is a left p-injective ring. Therefore (b) implies (c).

(c) Since ${}_BB$ is YJ-injective, then every non-zero-divisor is invertible in B which implies that B coincides with its classical left (and right) quotient ring. By a well-known theorem of A. W. Goldie, B is simple Artinian.

(d) If B is right YJ-injective, then B is Artinian as in (c).

In any case, B must be Artinian. If A is prime, then A is simple Artinian as just seen. If A is not prime, since any proper prime factor ring of A is Artinian, by [5, Lemma 18.34B], A must be left Artinian. We have proved that (2) implies (1).

Finally, we give a "test module" for a ring to be strongly regular with non-zero socle. □

Theorem 4. *The following conditions are equivalent:*

(1) *A is strongly regular with non-zero socle;*

(2) *A contains a finitely generated reduced YJ-injective maximal left ideal.*

Proof. (1) implies (2) evidently.

Assume (2). Since A contains a finitely generated YJ-injective maximal left ideal M which is reduced, then A is a reduced ring [33, Lemma 2]. Let $0 \neq b \in M$. There exist a positive integer n such that any left A-homomorphism of Ab^n into M extends to A. If $j : Ab^n \to M$ is the natural inclusion, there exist $y \in M$ such that $b^n = j(b^n) = b^n y$. Now $1 - y \in r(b^n) = r(b)$ (because A is reduced). Then $b = by \in bM$ which proves that ${}_AA/M$ is flat [4, p.458]. Now ${}_AA/M$ is finitely related flat which is therefore projective. Let $A = M \oplus U$, where U is a minimal left ideal of A. Since A is reduced, then ${}_AU$ must be injective. Since ${}_AM$ is YJ-injective, then $A = M \oplus U$ is left YJ-injective. Therefore A is a reduced left YJ-injective ring which is then strongly regular by [30, Proposition 1(2)]. Thus (2) implies (1). □

We are unable to answer the following questions.

Question 2. Is a MELT fully idempotent right p-injective ring VNR ? (MELT fully idempotent rings need not be VNR [36]).

Question 3. If A contains a reduced p-injective maximal left ideal, is A strongly regular ?

Acknowledgement

I would like to thank the referee for helpful comments and suggestions leading to this improved version of the paper.

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