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SOME GEOMETRIC PROPERTIES OF SPECIAL DOMAINS IN A BANACH SPACE

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1. Introduction

The Hartogs domains and Reinhardt domains are classical subjects of complex analysis in several variables. They have been investigated since the beginning of the 20th century. In particular, much attention has been given to properties of these domains from the viewpoint of hyperbolic analysis since S. Kobayashi introduced the notion of the Kobayashi pseudodistance and used it to study geometric function theory in several complex variables.

In 1981 Kerzman and Rosay [7] and Sibony [17] studied the complete hyperbolicity of the Hartogs domain $\Omega_\varphi(\Delta)$, where Δ is the open unit disc in $\mathbb{C}$. In 2000 Thai and Duc [19] gave a sufficient condition for the complete hyperbolicity of the Hartogs domain $\Omega_\varphi(X)$, where X is a complex space. Unfortunately, this condition is not explicit.

The first aim of this note is to give more explicit conditions for the complete hyperbolicity of $\Omega_\varphi(X)$, where X is a Banach analytic space.

Up to now, as far as we know, there are the following three classes of (finite dimensional) complex manifolds having (PEP):

a) Every Siegel domain of the second kind in $\mathbb{C}^n$ [16],

b) Every hyperbolic compact Riemann surface [15],

c) Every compact manifold whose universal covering is a polynomially convex bounded domain of $\mathbb{C}^n$ [18].

The second aim of this paper is to show the new class of Banach analytic spaces which also have (PEP). That is the Hartogs domain $\Omega_\varphi(X)$, where X is a Banach analytic space.

In [13], Jarnicki and Pflug gave necessary and sufficient conditions on the $O^{(>0)}$-domain of holomorphy of pseudoconvex Reinhardt domains in $\mathbb{C}^n$. Ealier, in [11] Pflug showed that every bounded balanced pseudoconvex Reinhardt domain in $\mathbb{C}^n$ is finitely complete Caratheodory and thus is a $O^{(>0)}$-domain of holomorphy.

The last aim of this paper is to generalize the above-mentioned results to the case of Reinhardt domains in a Banach space.

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Here is a brief outline of the content of this paper. In §2 we review some basic notions needed for our purpose. In §3, §4, and §5 we are going to prove the following results.

Theorem A. *Let X be a Banach analytic space and φ an upper-semicontinuous function on X.*

(i) *If $\Omega_\varphi(X)$ is complete hyperbolic, then φ is continuous.*

(ii) *Let X be complete hyperbolic and φ satisfy the following condition: For each $x \in X$, there exists a neighbourhood V of x such that for every $\varepsilon > 0$, there exist functions $h_1, \dots, h_n$ holomorphic on V for which*

a) $h_j(x') \neq 0, \ \forall x' \in V, \ \forall j = 1, \dots, n;$

b) $\varphi(x') - \varepsilon \leq \max_{1 \leq j \leq n} \{log|h_j(x')|\} \leq \varphi(x'), \ \forall x' \in V.$

Then $\Omega_\varphi(X)$ is complete hyperbolic.

Theorem B. *Let X be a Banach analytic space and $\varphi : X \longrightarrow [-\infty; +\infty)$ an upper-semicontinuous function on X. Then $\Omega_\varphi(X)$ has* (PEP) *if and only if X has* (PEP) *and $\varphi(x) > -\infty$ for all $x \in X$.*

Theorem C. *Let Ω be a balanced pseudoconvex Reinhardt domain in a Banach space B with an unconditional basis $\{e_n\}_{n=1}^{\infty}$ such that the gauge functional h_Ω is continuous. Then Ω is a $O^{(>0)}$-domain of holomorphy.*

2. Basic notions

We shall make use of properties of Banach analytic spaces in Mazet [10] and properties of the Kobayashi pseudodistance on Banach analytic spaces in Kobayashi [8] or Franzoni and Vesentini [4].

2.1. We denote the Kobayashi pseudodistance on a Banach analytic space X by d_X. A complex space X is said to be hyperbolic if d_X is a distance defined the topology of X. If X is Cauchy complete for d_X, we say that X is complete hyperbolic. It is known [4] that every infinite dimensional Banach analytic space contains a domain D such that d_D is a distance but it does not define the topology of D. Moreover [8], every finite dimensional Cauchy complete hyperbolic space is finitely complete, i.e. every ball in X is relatively compact.

2.2. Let X be a Banach analytic space. A plurisubharmonic function φ on X is an upper-semicontinuous function

$$\varphi : X \longrightarrow [-\infty, +\infty),$$

such that $\varphi \circ \sigma$ is either subharmonic or $-\infty$ for every holomorphic map $\sigma : \Delta \to X$, where Δ is the open unit disc in $\mathbb{C}$.

2.3. A subset S of an open subset Z of a Banach space B is said to be pluripolar if for every $x \in S$ there exist a neighbourhood U of x and a plurisubharmonic function φ on U such that $\varphi\big|_{U \cap S} = -\infty$.

2.4. A Banach analytic space X is called to have the holomorphic extension property through closed pluripolar sets ((PEP) for short) if every holomorphic map

$$f : Z \setminus S \longrightarrow X,$$

extends holomorphically over Z, where S is a closed pluripolar subset of a domain Z of a Banach space B.

2.5. Let B be a Banach space and $\{e_n\}_{n=1}^{\infty} \subset B$. We say that $\{e_n\}_{n=1}^{\infty}$ is an unconditional basis of B if $\{e_n\}_{n=1}^{\infty}$ is a Schauder basis of B and for all $x \in B$, the series $\sum\limits_{n=1}^{\infty} e_n^*(x)e_n$ is unconditional convergent to x, where $\{e_n^*\}_{n=1}^{\infty}$ denotes the sequence of coefficient functionals of $\{e_n\}_{n=1}^{\infty}$. A domain Ω in B is said to be a Reinhardt domain if

$$\sum_{n=1}^{\infty} e^{i\theta_n} e_n^*(x)e_n \in \Omega$$

for all $x = \sum\limits_{n=1}^{\infty} e_n^*(x)e_n \in \Omega$ and all $\{\theta_n\}_{n=1}^{\infty} \subset \mathbb{R}$.

2.6. Let φ be an upper-semicontinuous function on a Banach analytic space X. Define

$$\Omega_{\varphi}(X) = \{(x, \lambda) \in X \times \mathbb{C} : |\lambda| < e^{-\varphi(x)}\} \subset X \times \mathbb{C}.$$

The domain $\Omega_{\varphi}(X)$ is called a Banach Hartogs domain.

2.7. Let B be a Banach space and $\delta_0(x) := \left(1 + \|x\|^2\right)^{-\frac{1}{2}}$, $x \in B$. For every domain $G \subset B$, put $\delta_G := \min\{\rho_G, \delta_0\}$, where ρ_G denotes the Euclidean distance to $B \setminus G$. For $N \geq 0$, let

$$O^{(N)}(G, \delta_G) := \left\{ f \in O(G) : \|\delta_G^N \cdot f\|_{\infty} < +\infty \right\}$$

be the space of all holomorphic functions with polynomial growth in G of degree $\leq N$ ($\|\cdot\|_{\infty}$ denotes the supremum norm). The domain G is said to be of type $O^{(>0)}(G \in O^{(>0)})$ if for each $N > 0$, G is an $O^{(N)}(G, \delta_G)$ - domain of holomorphy.

3. Proof of Theorem A

Lemma ([3], [1]). *Let $\theta : X \to Y$ be a holomorphic map between Banach analytic spaces. If Y is complete hyperbolic and for each $y \in Y$ there exists a neighbourhood V of y such that $\theta^{-1}(V)$ is complete hyperbolic, then X is complete hyperbolic.*

Proof. We first show that X is hyperbolic. Let $\{x_n\} \subset X$ and $d_X(x_n, x_0) \to 0$, $x_0 \in X$. We must prove that $x_n \to x_0$. Since Y is hyperbolic and $d_Y(\theta x_n, \theta x_0) \leq$

$d_X(x_n, x_0)$, it follows that $\{\theta x_n\}$ converges to θx_0. Put $\theta x_0 = y_0$. By the hypothesis, we can find a neighbourhood V of y_0 such that $\theta^{-1}(V)$ is hyperbolic. On the other hand, since d_Y defines the topology of Y, there exists a neighbourhood W of y_0 such that $d_Y(W, \partial V) > 0$. Thus there exists $\delta > 0$ such that $f(\delta\Delta) \subset V$ for every holomorphic map f from Δ into Y such that $f(0) \in W$, where Δ denotes the open unit disc in $\mathbb{C}$. We may assume that the neighbourhood W has the from

$$W = U(y_o, r) = \{y \in Y : d_Y(y_0, y) < r\}$$

and $x_n \in \theta^{-1}(W)$ for all $n \geq 1$.

Put $W' = U(y_0, r/2)$. To prove that $d_{\theta^{-1}(W)}(x_n, x_0) \to 0$ and hence $x_n \to x_0$, we only need to show that there exist positive numbers c, s such that

$$(*) \qquad d_X(p, q) \geq \min\left\{s, c d_{\theta^{-1}(W)}(p, q)\right\}, \quad \text{for all } p, q \in \theta^{-1}(W').$$

Consider a holomorphic chain joining p and q: $\{f_i\}_{i=1}^k$, $f_i : \Delta \to X$ are holomorphic, $f_i(0) = p_{i-1}$, $f_i(a_i) = p_i$, $i = 1, \ldots, k$, where $p_0 = p$; $p_k = q$; $a_1, \ldots, a_k \in \Delta$.

There are only two cases:

(1) $p_j \notin \theta^{-1}(W')$ for some $j = 1, \ldots, k$. We have

$$\begin{aligned}\sum_{i=1}^k d_\Delta(0, a_i) &\geq \sum_{i=1}^k d_X\big(f_i(0), f_i(a_i)\big) \\ &\geq \sum_{i=1}^k d_Y\big(\theta f_i(0), \theta f_i(a_i)\big) \\ &\geq d_Y\big(y_0, \theta f_j(a_j)\big) \geq r/2.\end{aligned}$$

(2) $p_0, \ldots, p_k \in \theta^{-1}(W')$. Then $\theta f_i(\delta\Delta) \subseteq V$ for all $i = 1, \ldots, k$. If $a_j \notin (\delta/2)\Delta$ for some $j = 1, \ldots, k$. Then

$$\sum_{i=1}^k d_\Delta(0, a_i) \geq d_\Delta(0, \delta/2).$$

If $a_i \in (\delta/2)\Delta$ for $i = 1, \ldots, k$, then there is $c > 0$ such that

$$d_\Delta(y, z) \geq c d_{\delta\Delta}(y, z) \quad \text{for all } y, z \in (\delta/2)\Delta.$$

Thus

$$\begin{aligned}\sum_{i=1}^k d_\Delta(0, a_i) \geq c\sum_{i=1}^k d_{\delta\Delta}(0, a_i) &\geq c\sum_{i=1}^k d_{\theta^{-1}(W)}(f_i(0), f_i(a_i)) \\ &= c\sum_{i=1}^k d_{\theta^{-1}(W)}(p_i, p_{i-1}) \\ &\geq c d_{\theta^{-1}(W)}(p, q).\end{aligned}$$

So there exist c, $s > 0$ with the required property.

Finally, we prove that X is complete hyperbolic. Let $\{x_n\}$ be a Cauchy sequence in X. It is easy to see that $\{\theta x_n\}$ is also a Cauchy sequence in Y. We may assume that $\{\theta x_n\}$ converges to $y_0 \in Y$. By the hypothesis, we can find a neighbourhood V of y_0 such that $\theta^{-1}(V)$ is complete hyperbolic. We let $W = U(y_0, r) \subset V$. Without loss of generality we can assume that $x_n \in \theta^{-1}(W')$ for every $n \geq 1$, where $W' = U\Big(y_0, \dfrac{r}{2}\Big)$. Since $\{x_n\}$ is a Cauchy sequence, there exists $n_0 \geq 1$ such that $d_X(x_m, x_n) < s$ for all $m, n \geq n_0$. By the inequality (*), it implies that

$$\begin{aligned} d_X(x_m, x_n) &\geq c d_{\theta^{-1}(W)}(x_m, x_n) \\ &\geq c d_{\theta^{-1}(V)}(x_m, x_n) \quad \text{for all } m, n \geq n_0. \end{aligned}$$

This implies that $\{x_n\}$ is a Cauchy sequence in a complete hyperbolic space $\theta^{-1}(V)$. Hence $\{x_n\}$ converges to a point in $\theta^{-1}(V)$. □

We now prove Theorem A.

(i) Assume that $\Omega_\varphi(X)$ is Cauchy complete hyperbolic but φ is not continuous at $x_0 \in X$. Since φ is upper semicontinuous, we can find a sequence $\{x_k\} \subset X$ which converges to x_0 such that

$$e^{-\varphi(x_0)} < r < s < e^{-\varphi(x_k)} \quad \text{for } k \geq 1.$$

Let $\lambda_0 = \dfrac{e^{-\varphi(x_0)}}{r}$, i.e., $|r\lambda_0| = e^{-\varphi(x_0)}$. Then $(x_0, r\lambda_0) \notin \Omega_\varphi(X)$. Choose $\lambda > 0$ such that

$$|r\lambda| < e^{-\varphi(x_0)},$$

and choose $\alpha > 0$ such that

$$|r\lambda| = e^{-\varphi(x_0) - \alpha}.$$

Take $\delta > 0$ such that $e^{-\varphi(x_0)-\alpha} \leq e^{-\varphi(x_k)}$, $\forall \ \|x - x_0\| < \delta$. We have

$$\begin{aligned} &d_{\Omega\varphi(X)}\big((x_k, r\lambda_0), (x_j, r\lambda_0)\big) \\ &\qquad \leq d_{\Omega\varphi(X)}\big((x_k, r\lambda_0), (x_0, r\lambda_0)\big) + d_{\Omega\varphi(X)}\big((x_0, r\lambda_0), (x_j, r\lambda_0)\big) \\ &\qquad \leq d_{B(x_0,\delta)}(x_k, x_0) + d_{B(x_0,\delta)}(x_0, x_j). \end{aligned}$$

Thus $\big\{(x_k, r\lambda_0)\big\}$ is a Cauchy sequence in $\Omega_\varphi(X)$, but $\big\{(x_k, r\lambda_0)\big\}$ converges to $(x_0, r\lambda_0) \notin \Omega_\varphi(X)$.

(ii) Consider the canonical projection

$$\begin{aligned} \pi : \quad \Omega_\varphi(X) &\longrightarrow X \\ (x, \lambda) &\longmapsto x. \end{aligned}$$

For every $x \in X$, there exists a neighbourhood V of x such that $\forall \varepsilon > 0$, $\exists \ h_1, \dots, h_n \in H(V)$ such that

a) $h_j(x) \neq 0$, $\ \forall j = 1, \dots, n$; and $\forall x \in V$;

b) $\varphi(x) - \varepsilon < \max\big\{\log|h_j(x)| : 1 \leq j \leq n\big\} < \varphi(x)$.

Without loss of generality we can assume that $V = B(x_0, \delta)$, $\delta > 0$. Choose $\varepsilon_k \downarrow 0$. By the hypothesis, for each $k \geq 1$, we can find $h_j^k \in H(V)$, $j = 1, 2, \dots, n_k$, such that $h_j^k(x) \neq 0$ for every $x \in V$, and

$$\varphi(x) - \varepsilon_k < \max\left\{\log|h_j^k(x)| : j = 1, 2, \dots, n_k\right\} < \varphi(x), \quad \forall x \in V.$$

For each k there exists $1 \leq j_k \leq n_k$ such that

$$\varphi(x_k) - \varepsilon_k \leq \log|h_{j_k}^k(x_k)| < \varphi(x_k)$$

or

$$e^{\varphi(x_k) - \varepsilon_k} \leq |h_{j_k}^k(x_k)| < e^{\varphi(x_k)}.$$

Put

$$f_k(x, \lambda) = h_{jk}^k(x)\lambda, \quad \text{for } (x, \lambda) \in \pi^{-1}(V).$$

Since

$$\pi^{-1}(V) \subset \Big\{(x, \lambda) : |\lambda| < e^{-\varphi(x)}, x \in V\Big\},$$

we have

$$|f_k(x, \lambda)| = |h_{j_k}^k(x)|\ |\lambda| < e^{\varphi(x)} e^{-\varphi(x)} = 1, \quad \forall x \in V.$$

Hence

$$\sup_{\pi^{-1}(V)} |f_k| \leq 1, \quad \text{for } k \geq 1.$$

Obviously, $f_k(x, 0) = 0$ for $x \in V$ and $k \geq 1$.

Now we prove that $\pi^{-1}(V)$ is complete hyperbolic.

Assume that $\{(x_k, \lambda_k)\} \subset \pi^{-1}(V)$ is a Cauchy sequence for $d_{\pi^{-1}(V)}$. Since $\pi^{-1}(V)$ is bounded, it follows that $\{(x_k, \lambda_k)\}$ is a Cauchy sequence in B. Hence $(x_k, \lambda_k) \to (x_0, \lambda_0) \in \overline{\pi^{-1}(V)}$.

Assume that $(x_0, \lambda_0) \in \partial\pi^{-1}(V)$, i.e., $|\lambda_0| = e^{-\varphi(x_0)}$. We have

$$\begin{aligned}\lim_{k\to\infty} d_{\pi^{-1}(V)}\big((x_k, \lambda_k), (x_k, 0)\big) &\geq \lim C_{\pi^{-1}(V)}\big((x_k, \lambda_k), (x_k, 0)\big) \\ &\geq \lim\ \log \frac{1 + |f_k(x_k, \lambda_k)|}{1 - |f_k(x_k, \lambda_k)|} = +\infty,\end{aligned}$$

where $C_{\pi^{-1}(V)}$ denotes the Caratheodory distance of $\pi^{-1}(V)$. This is impossible. Hence $(x_0, \lambda_0) \notin \partial\pi^{-1}(V)$, i.e., $\{(x_k, \lambda_k)\}$ is the convergent sequence in $\pi^{-1}(V)$.

Remark. There exists a continuous plurisubharmonic function φ in $\Delta_R^2 = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1| < R,\ |z_2| < R\}$ for some $R > 0$ such that $\Omega_\varphi(\Delta_R^2)$ is not complete hyperbolic.

Let g be the continuous logarithmically-plurisubharmonic function in $\mathbb{C}^2$ which constructed by M. Jarnicki and P. Pflug [14]. Then $\{z \in \mathbb{C}^2 : g(z) < 1\}$ is bounded and has a connected component Z such that Z is not complete hyperbolic [14]. Choose $R > 0$ such that $\{z \in \mathbb{C}^2 : g(z) < 1\} \subset \Delta_R^2$. Consider the Hartogs domain

$\Omega_\varphi(\Delta^2)$, where $\varphi = \log g$. Since $\{(z,1) : z \in Z\}$, it follows that $\Omega_\varphi(\Delta^2_R)$ is not complete hyperbolic.

4. Proof of Theorem B

($\Rightarrow$) Assume that $\Omega_\varphi(X)$ has (PEP).

Since X is contained in $\Omega_\varphi(X)$ as a closed Banach analytic subspace, it follows that X has (PEP). Since X contains no complex lines, $\varphi(x) > -\infty$ for all $x \in X$.

It remains to show that φ is plurisubharmonic. Given $\sigma : \Delta \to X$ is a holomorphic map. In order to prove the subharmonicity of $\varphi \circ \sigma$ it suffices to check that $\Omega_{\varphi\circ\sigma}(\Delta)$ is pseudoconvex [6].

Assume that $g = (g_1, g_2) : \Delta^* \to \Omega_{\varphi\circ\sigma}(\Delta)$ is holomorphic, where $\Delta^* = \Delta \setminus \{0\}$. Extend g_1 to a holomorphic map $\hat{g}_1 : \Delta \to \Delta$. Consider the holomorphic map $\theta : \Omega_{\varphi\sigma}(\Delta) \to \Omega_\varphi(X)$ given by

$$\theta(x,\lambda) = (\sigma(x), \lambda) \quad \text{for} \quad (x,\lambda) \in \Omega_{\varphi\sigma}(\Delta).$$

Since $\Omega_\varphi(X)$ has the (PEP), $f = \theta \circ g$ can be extended to a holomorphic map $\hat{f} = (\hat{f}_1, \hat{f}_2) : \Delta \to \Omega_\varphi(X)$. By the relation $\hat{f}_1 \circ \sigma = g_1$, it follows that $\hat{f}_1 \circ \sigma = \hat{g}_1$. Thus the form

$$\hat{g}(x) = \big(\hat{g}_1(x), \hat{f}_2(x)\big) \quad \text{for } x \in \Delta,$$

defines a holomorphic extension of g. Since $\Omega_{\varphi\sigma}(\Delta)$ is a domain in $\mathbb{C}^2$, it follows that $\Omega_{\varphi\sigma}(\Delta)$ is pseudoconvex.

($\Leftarrow$) Now assume that X has (PEP) and φ is plurisubharmonic on X with $\varphi(x) > -\infty$ for all $x \in X$. Suppose that

$$f = (f_1, f_2) : Z \setminus S \longrightarrow \Omega_\varphi(X)$$

is a holomorphic map, where Z is an open set in a Banach space B and S is a closed pluripolar subset of Z. By [2], we may assume that $B \cong \mathbb{C}^n$. By the hypothesis, f_1 can be extended to a holomorphic map

$$\hat{f}_1 : Z \longrightarrow X.$$

Assume that x_0 is an arbitrary point of S. Since $\varphi(x_0) > -\infty$, it follows from [6] that $e^{-a\varphi}$ is integrable at x_0 for all $a > 0$. Choose a neighbourhood U of x_0 such that

$$\int_U e^{-3\varphi_0 f(x)} dx < +\infty.$$

Since $|f_2(x)|^3 < e^{-3\varphi_0 f(x)}$ for all $x \in U \setminus S$, it follows that $f_2 \in L_3(U)$. On the other hand, since $\lambda_{2n-\frac{3}{2}} = 0$ [9], where $\lambda_\alpha(E)$ denotes the α-dimensional Hausdorff measure of E, $\alpha > 0$, f_2 can be extended to a holomorphic function $\hat{f}_2$ on U by [5]. Hence f can be extended to a holomorphic map $\hat{f} : U \to X \times \mathbb{C}$. Since $\log|f_2(x)| + \varphi(f_1(x)) < 0$ for $x \in U$, by the maximum principle, we have

$$\log|\hat{f}_2(x)| + \varphi(\hat{f}_1(x)) < 0 \quad \text{for } x \in U.$$

Thus $\hat{f} : U \longrightarrow \Omega_\varphi(X)$. Since x_0 is arbitrary, f is extended to a holomorphic map from Z into $\Omega_\varphi(X)$.

5. Proof of Theorem C

(i) Let $z_0 \in \partial\Omega$ and $\varepsilon > 0$. Consider the cone

$$V = \Big\{ tz \Big| t > 0 \text{ and } z \in \partial\Omega \text{ such that } \|z - z_0\| < \varepsilon \Big\}.$$

By the continuity of h_Ω, it is easy to see that V is an open neighbourhood of z_0.

Put $B_n = \operatorname{Span}(e_1, \ldots, e_n)$ for $n \geq 1$. Since $\bigcup\limits_{n=1}^{\infty} B_n$ is dense in B, there exists $z' \in V \cap B_n$ such that $\|z' - z_0\| < \varepsilon$. Writing $z' = tz$, $t > 0$ and $z \in \partial\Omega$ such that $\|z - z_0\| < \varepsilon$. We have $z \in B_n$. Thus $\bigcup\limits_{n=1}^{\infty} \partial(\Omega \cap B_n)$ is dense in $\partial\Omega$.

(ii) Let $z_0 \in \bigcup\limits_{n=1}^{\infty} \partial(\Omega \cap B_n)$. Take n such that $z_0 \in \partial(\Omega \cap B_n)$. Given $N > 0$, by [12] there exists $g \in O^{(N)}(\Omega \cap B_n, \delta_{\Omega\cap B_n})$ such that g cannot be extended holomorphically to z_0. Then $f = g \cdot \pi_n \in O(\Omega)$ and f cannot be extended holomorphically to z_0. Moreover, $f \in O^{(N)}(\Omega, \delta_\Omega)$ because

$$\rho(z, B \setminus \Omega) \leq \rho(z, B \setminus \pi_n^{-1}(\Omega \cap B_n)) = \rho(\pi_n z, B_n \setminus \Omega \cap B_n)$$

for $z \in \Omega$.

(iii) Choose a countable dense subset $\{z_n\}$ of $\bigcup\limits_{n \geq 1} \partial(\Omega \cap B_n)$ and a sequence $\varepsilon_n \downarrow 0$. For $n, m \geq 1$ consider the Banach space $F_{n,m}$ given by

$$F_{n,m} = \Big\{ f \in O(\Omega \cup B(z_n, \varepsilon_m)) : f\big|_\Omega \in O^{(N)}(\Omega, \delta_\Omega), \big\|f\big\|_{B(z_n,\varepsilon_m)} < \infty \Big\}.$$

Let $R_{n,m} : F_{n,m} \longrightarrow O^{(N)}(\Omega, \delta_\Omega)$ be the restriction map. Then $\operatorname{Im} R_{n,m} \neq O^{(N)}(\Omega, \delta_\Omega)$ for $n,\ m \geq 1$. By the Baire theorem, $\bigcup \operatorname{Im} R_{m,n} \neq O^{(N)}(\Omega, \delta_\Omega)$. Thus there exists $f \in O^{(N)}(\Omega, \delta_\Omega)$ which cannot be extended holomorphically through every point of $\partial\Omega$.

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